# FINITE ELEMENT MESH SIZE IN FATIGUE LIFE EFFECT Muhanad Hamed Mosa

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# ABSTRAC

Pipe elbows are consequential parts in a piping system. When a thin-wall elbow is exposed to variable amplitude loading as internal cyclic pressure, piping elbows are vulnerable to crack at the crown. This paper presents a technique to predict the fatigue life of elbows with the application of variable amplitude loadings (VAL). For this goal, the finite element analysis technique was being used for the modeling and simulation. The fatigue life prognostication was carried out using the finite element based fatigue analysis codes. Numerical life prognostication results of three types of elbows (135°, 45° and curved ) under VAL are presented and discussed. Varying elbows shows different fatigue life behavior, the curved one gives better life. The two types of elements (Tet4 and Tet10) were investigated with optimum mesh size to get the maximum principle stress. The simulation results showed that more studies on the piping elbows necessity to be performed in order to obtain more delicate fatigue life.

**Keywords:** Fatigue life, Finite element analysis (FEA), Pipe elbow, Variable amplitude

loading(VAL), Tetrahedral element(Tet).

#### الخلاصة

تعتبر الانابيب الكوعية من العناصر الهامة في الانظمة الخاصة بشبكات الانابيب ، فعندما يتعرض الكوع الرقيق الجدران لاحمال متغيرة السعة كالاختلافات الدورية في الضغط الداخلي لتلك الانابيب يكون عرضة الى نشوء صدع في الانبوب . يقدم هذا البحث تقنية للتنبؤ بعمر الانبوب الكوعي حيث شمل البحث مدى واسع من الاحمال المتغيرة حيث تم استخدام تقنية تحليل العناصر المحدودة للنمذجة والمحاكاة. وقد تم تنفيذ هذه الطريقة باستخدام ثلاث انواع من الزوايا لتلك الانابيب ( ٤٥ درجة ، ١٣٥ درجة والانابيب المنحنية) و تم اجراء الدراسة والمناقشة للنتائج لهذه الانواع الثلاثة، ولقد تبين بان اختلاف الزوايا والشكل في الانابيب الكوعية يؤدي الى الاختلاف في اعمار هذه الانواع الثلاثة، ولقد تبين بان اختلاف الزوايا والشكل في الانابيب الكوعية يؤدي الى الاختلاف في اعمار هذه الانابيب اعتمادا على انواعها ولقد تبين من خلال هذه الدراسة بان افضلها عمرا هي الانابيب المنحنية .عند اجراء النمذجة تم فحص هذين النوعين من عناصر ( Tetl و المثل الشبكة الخاصة بالعناصر المحددة وذلك للحصول على أقصى قدر من الاجهاد الاساسي او الاجهاد الاقصى ولقد واظهرت نتائج المحاكاة ضرورة اجراء المزيد من الدراسات على الأنابيب الكوعية وذلك من أجل التشبكة الخاصة بالعناصر المحددة وذلك للحصول على أقصى قدر من الاجهاد الاساسي او الاجهاد الاقصى ولقد وأظهرت نتائج المحاكاة ضرورة اجراء المزيد من الدراسات على الأنابيب الكوعية وذلك من أجل الحصول على منائج اكثر دقة للتنبؤ بحياة تلك الإنابيب.

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#### INTRODUCTION

Piping elbows are one of the critical components of the piping systems in all applications. Elastic–plastic fracture analysis of piping [1] is increasingly important in life and structural integrity assessments. Significant efforts in developing fracture mechanics methodology have been made during the last three decades [6], together with validation against finite element results and experimental pipe test data. Accurate stress analyses of these components are required for reliable prediction of strength and residual life. Analytical solutions are available for only a few idealized geometries. When a thin-walled (diameter to thickness ratio >10) elbow is subjected to an in-plane moment, ovalisation of the circular section into an elliptical one, introduces high bending stress (circumferential component) at the crown and a crack may initiate at the crown. This has been confirmed by analytical [4] and experimental work [5] It was noticed during an experiment by Naoki [7] that, for a stainless steel elbow with an axially through-wall crown crack, the crack opened at the outside surface and contacted at the inside surface, when subjected to in-plane closing bending moment.

The general trend given by the Goodman relation is one of decreasing fatigue life with increasing mean stress for a given level of alternating stress. The relation can be plotted to determine the safe cyclic loading of a part; if the coordinate given by the mean stress and the alternating stress lies under the curve given by the relation, then the part will survive. If the coordinate is above the curve, then the part will fail for the given stress parameters [2].

The failure criterion based on J-integral [8] is commonly used in elastic–plastic fracture mechanics. For estimation purposes, it is convenient to represent the J-integral as the sum of elastic and plastic components. The analytical, experimental and computational studies on this subject indicate that the J-integral and crack opening displacement are the most viable fracture parameters for characterizing crack initiation, crack growth and subsequent instability in ductile materials [9]. For arbitrary geometries, exact solutions are not available because of the complex nature of stresses and crack propagation at surface cracks in such cases. In this paper, finite element analysis of pipe elbows was carried out for different types of elbows (curved, 45<sup>0</sup> and 135<sup>0</sup>). A fatigue life analysis based on the stress life approach was followed to obtain an estimate of the expected life of elbows, with application of variable amplitude loadings. The full pipe elbow was modeled using tetrahedral element (Tet4 and Tet10). Different mesh size was used to see its effect on maximum principle stresses as well as the difference between the two element types.

### THE MATHEMATICAL MODEL

Many numbers of equations have been developed to describe the sigmoidal da/dN–dK relationship. In this paper the Austen growth model is known as the implicitly model threshold has been used and it is expressed in the following equation:

 $da/dN = C. (\Delta K_{eff})^{n}$ ....(1) Where  $\Delta K_{eff} = \Delta K_{max,eff} - \Delta K_{min,eff}$   $\Delta K_{max,eff} = \Delta K_{max} + K_{SF}$   $\Delta K_{min,eff} = max(\Delta K_{min}, K_{CL})$ 

And  $K_{SF}$  defined as the modification for static fracture and  $K_{CL}$  is known as the stress intensity at the crack closure. Furthermore, Austen modelled the onset of fast fracture using the following expression

$$K_{SF} = \frac{K_{max}}{K_{1C} - K_{max}}$$

Austen also took into account the threshold and short cracks by applying a crack closure stress KCL which is expressed as follows:

$$K_{CL} = K_{max} - K_{max} \sqrt{\frac{a+I_o}{a}} + \frac{\Delta K_{th}}{1-R}$$

......(3)

 $I_{\rm o}$  is the smallest crack size that will propagate and is given by:

$$I_o = \frac{1}{\pi \left(\frac{K_{th}}{\Delta \sigma_o}\right)^2}$$

......(4)

where,  $\Delta \sigma_0$  is the un-notched fatigue strength and  $\Delta K_{th}$  is the threshold stress intensity and the threshold stress intensity is expressed as a bilinear function of the mean stress and the threshold stress intensity is expressed as a bilinear function of the mean stress

Austen model does not possess any explicit mean stress correction. Austen argued the irrelevance of this and attributed it to a manifestation of crack closure and retardation [6].

The Goodman relation can be represented mathematically as:

$$\frac{\sigma_a}{\sigma_{\rm fat}} = 1 - \frac{\sigma_m}{\sigma_{\rm ts}}$$

.....(5)

Where  $\sigma_a$  is the alternating stress,  $\sigma_m$  is the mean stress,  $\sigma_{fat}$  is the fatigue limit for completely reversed loading and  $\sigma_{ts}$  is the ultimate tensile strength of the material. The general trend given by the Goodman relation is one of decreasing fatigue life with increasing mean stress for a given level of alternating stress [1].

# FINITE ELEMENT APPROACH

For many years, fatigue analysis has been thought of as following the logic as illustrated in

Fig. 1.[10] In this overview, the three main input parameters, namely geometry, material and loading, are regarded to have similar functions. These parameters seem to be the main input to any software for modelling and simulation



FIGURE 1. A conventional schematic flow of the fatigue analysis process (the general durability process)

The start of a fatigue failure is a precisely local process and it is also one that depends on the dynamics of the system. the exact location where a crack is going to start, is a critical factor and a general distribution of parameters throughout the component is of secondary interest. This is strictly why the finite element analysis (FEA) is important in this particular discipline. With FEA, an analyst can choose any location within a model and concentrate on it. In fact, using FEA can give a tighter control over the move from general geometry and loading to the local parameters, and allows dynamic factors to be dealt with more analytically. The type of element used here is tetrahedral element (4 and 10 nodes) shown in Figure 2.



FIGURE.2 Finite Element Mesh ((a) Tet4, (b) Tet10, (c) Integration grid points of the Tet10 nodes)

The results of the maximum principle stresses are used for the subsequent fatigue life analysis, using the cyclic load as an internal pressure to the pipe elbows. While the material used in this analysis is ASTM A533 steel as shows in Table1.

Title	Value	
Tensile Strength, Ultimate	<u>550</u> - <u>690</u> MPa	
Tensile Strength, Yield	345 MPa	
Modulus of Elasticity	200 GPa	
Poissons Ratio	0.29	
Shear Modulus	80.0 GPa	

**TABLE 1. Mechanical properties of ASTM A533 steel** 

The Finite Element technique was used for modeling and simulation ,three type of geometry of the case study in three-dimension mesh showed in Figure 3.



FIGURE 3. Three types of elbows ((a) 1350, (b) 450, (c) curved)

# **RESULTS AND DISCUSSION**

The FEM analysis for the modelling and simulation based on ANSYS software .the element used it tetrahedral element. The Figure 4 showed the contour (image) of the stresses distribution (maximum principle stresses) on curve elbows for Tet10 mesh elements at a high load level.

The Tet10 mesh is presumed to represent a more accurate solution since Tet4 meshes are known to be dreadfully stiff [2].



FIGURE 4. Contour display of stresses in

The stress-life (S-N) curve in the log diagram the three types of elbows based on Goodman theory using Tet10 element which, showed the difference in fatigue life with respect to the curvature of the elbows. Figure 5 shows the relation between the life and maximum stress . the curve for  $135^{0}$  elbow gave minimum life while others gave maximum life .



FIGURE 5. Stress-life curves (S-N) for elbows

The meshing is based on the geometry, model topology, analysis objectives and engineering judgment. Each type of the meshes has some distorted elements cause error to the modeling in areas of elevated stress. In the design stage, these areas should be remeshed and further refined to check for solution convergence [9].

The auto Tet Mesh approach is a highly automated technique for meshing solid regions of the geometry. The Tet4 compared to the Tet10 (10 nodes tetrahedral) mesh using the same global mesh length for the same loading conditions. The comparison between two meshes is presented in Figure 6. The result shows that the Tet10 mesh predicted higher maximum Principle stresses than that the Tet4 mesh [7].



FIGURE 6. The effect of mesh size on maximum principle stresses (Tet4 and 10Tet.)

The FEM generated using the tetrahedral elements with various mesh size are shown in Table2 for Tet10 and Table 3 for Tet4.

The convergence of the stress was considered as the main criteria to select the mesh size. For the same mesh size (1.4 mm), the maximum percentage difference between the stress values observed between the two models is (the one with 9902 elements and other one with 11498 elements) is 64%. The maximum percentage difference between the mesh sizes (1.2 mm to 1.4 mm) is 95%. Thus, the mesh size (1.4) mm with (11498) elements were preferred to use for the finite element analysis due to the limitation of the computational and the storage capacity.

Mesh Size (mm)	No. of Elements	No. of Nodes	No. of D.O.F.	Max. Principle Stresses (MPa)
0.6	5337	68733	204776	116
0.7	4221	54594	162526	111
0.8	34257	43536	129433	109
0.9	27198	36872	109585	98
1.0	21674	31465	93996	95.5
1.1	18358	26694	79195	95.2
1.2	15663	23124	68435	94
1.3	13274	19426	57535	93.5
1.4	11498	18173	53789	89.1
1.5	9651	15569	46033	82.9
1.6	9032	13516	41665	82.4
1.7	7731	12433	36433	81.3
1.8	6974	11781	35322	78
1.9	6164	12442	32661	77.9
2.0	6037	11782	31333	77.4

 TABLE 2. Mesh Size of the Tet10 element with Maximum Principle Stresses

TABLE 3. Mesh Size of the Tet4 element with Maximum Principle Stresses

Mesh Size (mm)	No. of Elements	No. of Nodes	No. of D.O.F.	Max. Principle Stresses (MPa)
0.8	34255	11480	34018	66
0.9	27199	9120	26966	66
1.0	21673	7295	21458	65
1.1	18359	6179	18158	62.2
1.2	15662	5262	15511	61.5
1.3	13273	4461	13112	60.2
1.4	9902	3328	9761	60.7
1.5	9661	3266	9515	60.6
1.6	9039	3058	8911	60.5
1.7	7721	2615	7574	60.4
1.8	6488	2183	6374	59
1.9	6072	2068	5969	55.1
2.0	5773	1951	5678	54.2

# CONCLUSION

Many fatigue prediction theories was studied and the best is the Goodman theory for different loading, this research, which showed the maximum life in curved elbow while for the 135<sup>0</sup> elbow is the minimum life. These results must be taking into account in design of piping system. The result shows that the Tet10 mesh predicted higher maximum principle stresses than that the Tet4 mesh with same mesh size, but with more number of elements, nodes and degree of freedom.We conclude that, using the ideal number of elements not only the minimum mesh size, but the effective element type for the finite element analysis due to the limitation of the computational and the storage capacity.

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