Mixed Convection Heat Transfer Inside A vented Square Enclosure with Concentric Rotation Inner Cylinder

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ABSTRACT:

Mixed convection heat transfer for laminar air flow inside a vented square enclosure with rotation inner cylinder is studied numerically The bottom wall is heated at constant temperature (T_h) while the other walls and the surfaces of the inner cylinder are adiabatic. An external flow enters the square cavity through an opening located down the left vertical wall and exits from an opening located in the upper right wall. The conservation governing equations (continuity, momentum and energy are solved by finite element method using Flex PDE software package. Results of velocity curves, isotherms and average Nusselt number show the effect of the variables: Richardson number (Ri = 0,6,10), Reynolds number (Re = 20, 50, 100), Prandtl number (Pr = 0.7), the dimensionless angular velocity (Ω = 0 –3) and dimensionless radius of rotating cylinder (R= 0 - 0.3) on heat transfer and flow filed. The results show that, the average Nusselt number increases with increasing Ri, Re, R and decreases with increasing Ω. The results are compared with other authors in the literature and give a good agreement.

Key words: Mixed convection, Rotating cylinder, Square enclosure.

انتقال الحرارة بالحمل المختلط داخل فجوة مربعة تحوي اسطوانة داخلية متمركزة دوارة

المستخلص :

تم في هذا البحث دراسة نظرية لانتقال الحرارة بالحمل المختلط لجريان الهواء الطباقي داخل فجوة مربعة تحتوي اسطوانة داخلية دوارة. الجدار السفلي مسخن عند درجة حرارة ثابتة (T_h) أما الجدران الأخرى وكذلك سطح الاسطوانة الداخلية فتكون معزولة. الجريان الخارجي يدخل الفجوة المربعة بدرجة حرارة (T_{in}) خلال فتحة تقع أسفل الجدار الأيسر ويخرج من فتحة تقع في أعلى الجدار الأيمن. معادلات الحفظ الحاكمة (الاستمرارية ، الزخم

الطاقة) حُلت بطريقة العناصر المحددة باستخدام الحقيبة البرمجية FlexPDE . نتائج مخططات السرعة و خطوط ثابتة درجة الحرارة وضحت تأثير المتغيرات : رقم ريتشارد (Ri = 0,6,10)، رقم رينولدز (Re =20,50,100)، رقم براندل(Pr = 0.7), السرعة الزاوية اللابعدية (Ω=0-3) و نصف قطر الأسطوانة الدوارة اللابعدي (R=0-0.3) على انتقال الحرارة والجريان . وجد بان معدل رقم نسلت يزداد بزيادة Ri,Re ,R ويقل بزيادة Ω .النتائج قورنت مع نتائج باحثين آخرين وأظهرت نتائج جيدة.

1. Introduction:

Mixed convection occurs in many heat transfer devices, such as the cooling system of a nuclear power plant, large heat exchangers, cooling of electronic equipment and ventilation. The relative direction between the buoyancy force and the externally forced flow is important. In the case where the fluid is externally forced to flow in the same direction as the buoyancy force, the mode of heat transfer is termed assisting combined forced and natural convection. In the case where the fluid is externally forced to flow in the opposite direction to the buoyancy forced, the mode of heat transfer is termed opposing mixed convection [1]. There are many geometric configuration of enclosures filled with a convective fluids such as rectangular, triangular, elliptical and circular enclosures. Shung et. al. [2] studied numerically natural convection heat transfer in an enclosure with a rotating cylinder. A penalty finite-element method with a Net Jon-Raphson iteration algorithm is adopted to solve the governing equations with the boundary conditions. The results show that, when the value of (Gr/Re) is high, the enhancement of the heat transfer rate begins to be revealed. Khanafar et. al. [3] studied numerically mixed convection heat transfer in open ended enclosure for three different flow angles of attack. Discreization of the governing equations is achieved using a finite element scheme based on the Galerkin method of weighted residuals. The results show that thermal insulation of the cavity can be achieved through the use of high horizontal velocity flow. Lasode [4] studied numerically laminar mixed convection heat in vertical elliptic ducts containing an upward flowing fluid rotating about parallel axis. The coupled system of normalized conservation equations are solved using a power series expansion in ascending powers of rotational Rayleigh Number. The results show that, the mean Nusselt number is decreases with eccentricity for high heating rates.

Kim et. al. [5] performed numerically natural convection problem in a cooled square enclosure with an inner heated circular cylinder. The immersed boundary method was solved by finite volume method to simulate the flow and heat transfer over an inner circular cylinder in the Cartesian coordinates. The results show that, the location of the peak and valley of the local Nusselt number along the surfaces of the inner cylinder and enclosure depends on the location of the center of these vortices. Rahman et. al. [6] studied the mixed convection in a cavity contains a heat conduction of horizontal square block located at the center. The investigations are conducted for various values of geometric size, location and thermal conductivity of the block under Pr and Re. The results indicated that the average Nu number at the heated wall is highest for the lowest value of cavity aspect ratio ,but the average temperature of the fluid in the cavity and temperature at the cylinder center are the lowest for the highest value of aspect ratio. Costa et. al. [7] studied numerical mixed convection in a square enclosure with a rotating cylinder centered within. For high values of the cylinder radius, the overall Nusselt number is small if the rotating velocity is low, and it considerably increases, in a nearly linear way, with the rotating velocity absolute value and the overall Nusselt number becomes maximum if the diameter to height ratio is small, for rotating velocities close or equal to zero. Saha et. al. [8] studied numerically mixed convective flow in a lid-driven square cavity with uniform internal sources. The top moving lid of the cavity and the bottom wall is maintained at constant temperature, while the vertical walls are thermally insulated. The results show that, the average Nusselt number becomes an increasing function of increasing Ri.

Lee et. al. [9] investigated numerically the effect of the natural convection heat transfer in a square enclosure with a circular cylinder at different horizontal and diagonal locations the effect of buoyancy-induced convection on the fluid flow and heat transfer in the enclosure increases and as a result the locations of the local minimum of Nusselt number on the cylinder surface and the local peaks of the Nusselt number on the enclosure wall are strongly depended on the origin and the inclined direction of the thermal plume. Abood et. al. [10] studied numerically mixed convection heat transfer inside a vented square cavity with inner heated cylinder by finite element method to solve the conservation of governing equations. The results show that with increases of Re and Ri numbers the convective heat transfer becomes predominated over the conduction heat transfer. Alshara [11] studied numerically the effect of rotating horizontal single or multi cylinders on mixed convection heat transfer in an equilateral triangular enclosure filled with air. Three cases are performed: single rotating cylinder, three rotating cylinders at the same direction and three rotating cylinders at different directions. It was found that the average Nusselt number for the single or multi rotating cylinder is increased with increasing Ra, R and Ω for all cases.

The main aims of the present study are to show the effect of rotation inner cylinder on the mixed convection and to show the effect of the parameters: radius of cylinder, Reynolds number and Richardson number on Nusselt number, velocities, isotherms and average temperature.

2. Theoretical Analysis:

Figure 1. shows a schematic illustration of the problem under consideration. It is a two- dimensional vented square enclosure with height H and base L and centered rotating solid cylinder at angular velocity ω (clock wise is positive direction). The bottom wall is heated at constant temperature T_h while the other walls are assumed to be adiabatic. The inflow opening located on the bottom of the left vertical wall of the enclosure and exit from the opening located on the top right wall. The viscous dissipation term in the energy equation and radiation are neglected. The governing equations for the steady, laminar, two dimensional, incompressible flow with Boussinseq approximation and constant fluid properties can be written in non-dimensional form as follows [10]:

Continuity equation

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \qquad (1)$$

Momentum equation in x-direction

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P^*}{\partial X}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \qquad (2)$$

Momentum equation in y-direction

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P^*}{\partial Y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ri\,\theta \qquad (3)$$

Energy equation

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\mathrm{Re\ Pr}}\left[\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right] \qquad (4)$$

Where the dimensionless variables are defined as:

$$X=\frac{x}{L},\quad Y=\frac{y}{L},\quad U=\frac{u}{u_{in}},\quad V=\frac{v}{u_{in}},\ \theta=\frac{T-T_{in}}{T_h-T_{in}}$$

$$P^*=\frac{p}{\rho u_{in}^2},\quad \mathrm{Pr}=\frac{\nu}{\alpha},\quad Ri=\frac{Gr}{\mathrm{Re}^2}, Gr=\frac{g\beta(T_h-T_{in})L^3}{\nu^2}, and\quad \Omega=\frac{\omega(D/2)}{u_{in}}$$

The **Boundary Conditions are :**

$U = 1$, $V = 0$ and $\theta = 0$ at the inlet

$U = V = 0$ and $\theta = 1$ at bottom wall

$U = V = 0$ and $\frac{\partial \theta}{\partial N} = 0$ at the other walls of enclosure

$V = 0$, $P^* = 0$, $\frac{\partial U}{\partial X} = 0$ and $\frac{\partial \theta}{\partial X} = 0$ at the outlet

$U = \Omega (Y - Y_o)$, $V = -\Omega (X - X_o)$ and $\frac{\partial \theta}{\partial N} = 0$ at the surface of rotating solid cylinder

Where $N = \frac{n}{L}$, (X_o, Y_o) : the center of cylinder.

The average Nusselt number at the heated wall is calculated by[11]:

$$Nu_{av} = \int_0^1 - \left(\frac{\partial \theta}{\partial Y} \right)_{Y=0} dX \quad (5)$$

And the bulk average temperature is defined as:

$$\theta_{av} = \int \frac{\theta \, d\overline{V}}{\overline{V}} \quad (6)$$

Where $\overline{V}$: the volume of occupying fluid in square enclosure.

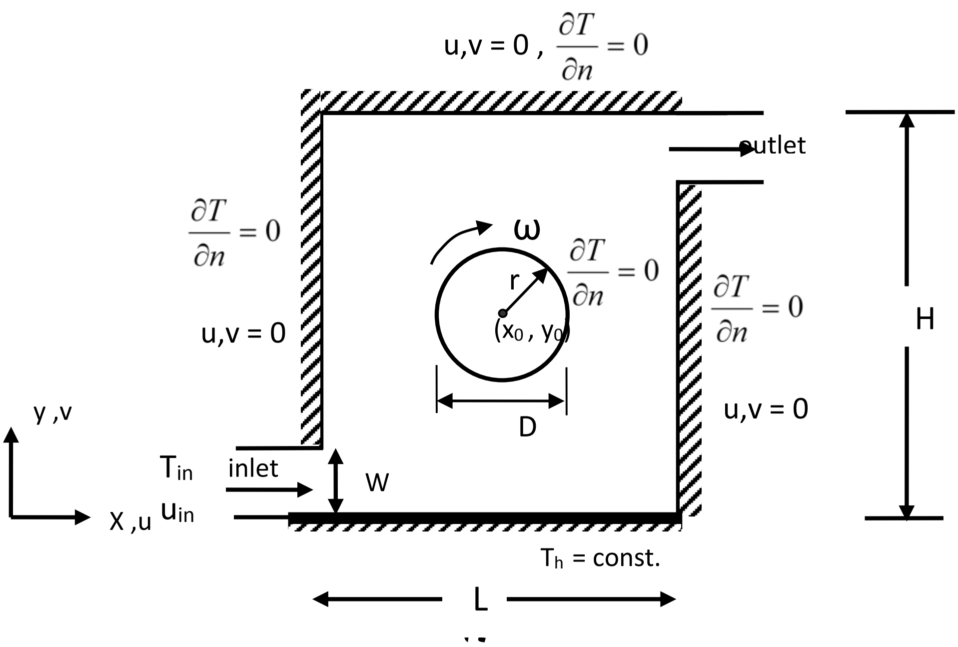


Fig.1: Sketch of the physical model and boundary condition

3. Numerical Solution :

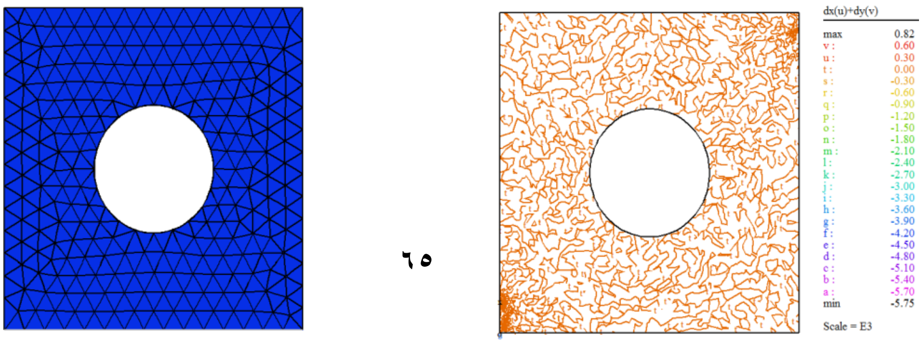
The governing equations (1) to (4) and the associated boundary conditions are solved numerically by finite element method using software package Flex PDE [12]. The continuity equation (1) is to be used as a constraint due to mass conservation and this constraint may be used to obtain the pressure distribution [13]. In order to solve the equations (2) to (4), we use a penalty parameter and the compressibility criterion by equation (1) which results in:

$$\nabla^2 P = \gamma \left(\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} \right) \tag{7}$$

γ is a penalty parameter that should be chosen either from physical knowledge or by other means [14]. A most convenient value for γ was attained in this study to be (10^{11} μ / L^2). The numerical solutions are obtained in terms of U- velocity, V- velocity and isotherms lines. The heat transfer coefficient in terms of average nusselt number along the heated surface (bottom wall) is obtained $Nu_{av} = = \int_0^1 - \left(\frac{\partial \theta}{\partial Y} \right)_{Y=0} dX$. Also, the bulk temperature is calculated from the relation $\theta_{av} = \int \frac{\theta \, d\bar{V}}{\bar{V}}$.

4. Validation and Comparison of The Study :

Geometry studied in this paper is an obstructed ventilated cavity; therefore several grid size sensitivity tests together with continuity equation ($\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0$) and obtained results showed an exactly validation of the velocity distribution for grid size obtained by imposing an accuracy of 10^{-3}. This accuracy is a compromised value between the result accuracy and the time consumed in each run. The grid domain for Ri = 1, Re = 50 is shown in (**Fig. 2-a**) and the distribution of ($\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y}$) over the domain is presented in (**Fig. 2-b**) .



(a) **(b)**

From the previous researches in the review, we could not be found research for mixed convection with rotating inner cylinder, therefore, to satisfy this a some reported data of Nu_{av} for the present work of mixed convection in square enclosure with stationary heated concentric cylinder are compared with work [10] as shown in table -1 which referred to the comparison of the present values of θ_{av} (second column) with the values of reference [10] (third column) at difference values of Ri. It is obvious that good agreement is obtained. As a result, the confidence in the present numerical solution is enhanced.

5. Results and Discussion :

The numerical results are represented by the isotherms lines , V-velocity, U-velocity, Nu_{av} and average temperature. The working fluid is air at Pr = 0.7. The ranges of the studied parameters are: Ri = 0-10 , Re = 20-100 , R = 0-0.3 and Ω = 0-3. **Fig. 3.** shows the effect of variation of radius of the rotational cylinder on V-velocity (left), U-velocity (middle) and isotherms (right) for Re = 50, Ri = 1, Pr = 0.7 and Ω = 1. The values of V-velocity and U-velocity are increased with increasing of R this is because of dominant the forced convection with reducing the region of the flow. It can be seen from the isotherms curves at R = 0 the growth of the thermal boundary layer is increased in the flow direction near the heated wall. As the radius of the inner cylinder is increased, this make a parallel lines of the isotherms which are covered the space between the inner cylinder wall and the surface of the vertical right wall of the enclosure. **Fig. 4.** shows the effect of the variation of rotational velocity Ω on V-velocity (left), U-velocity (middle) and isotherms (right) for Re = 50, Ri = 1, Pr = 0.7 and R = 0.2. The magnitude of V-velocity and U-velocity increases with increasing Ω because of the effect of the flow circulation. Also, this figure shows that the decrease in the growth of thermal boundary layer in the space between the cylinder and variation right wall increasing Ω. **Fig. 5.** shows the influence of Re number on V-velocity (left), U-velocity (middle) and isotherms (right) for Ri = 1, Pr = 0.7, Ω = 1 and R = 0.2. Both U- and V- velocities are increased with increasing Re number. From this figure, it can be seen that the growth of the thermal boundary layer is increased along the bottom wall of the enclosure. **Fig. 6.** shows the effect of variation of Ri number on V-velocity, U-velocity and isotherms for Re = 50, Ω = 1, Pr = 0.7 and R=0.2. This figure shows a plum like distribution of U- velocity & V-velocity at the inlet of the enclosure, it can be seen that the increase in Ri lead to increasing the values of U&V velocities. The temperature difference in the enclosure is increased with increasing of Ri due to the effect of the bouncy.

Fig. 7. (a) shows the effect of Ri number variation on Nu_{av} for Re = 50, Pr = 0.7, Ω = 1 and R = 0.2. As can be show the Nu_{av} is increased gradually with increasing of Ri number. This increments approximately linear due to the buoyancy effect. **Fig. 7. (b)** show the variation effect of Ri number on the average non dimensional temperature θ_{av} . Also, the value of θ_{av} is increased with increasing of Ri and the trend of the curve is the same as that in Fig. 7.(a). The effect of cylinder radius on Nu_{av} for Ri=1, Pr = 0.7, Ω = 1 and Re = 50 is shown in **Fig. 8. (a)**. It can be seen that, Nu_{av} increases linearly with increasing of cylinder radius due to the dominate of mixed convection with decreasing the cavity cross area. **Fig. 8. (b)** illustrate the variation of θ_{av} with cylinder radius for Ri = 1, Pr = 0.7, Ω = 1 and Re = 50. It can be observed that a small increases of θ_{av} with increasing of cylinder radius value from 0 to 0.15, this is may be attributed to the small effect of the secondary flow. When R > 0.15, it can be seen a deep variation of θ_{av} with R due to the decreases of the cavity cross section area which lead to increases the heat transfer process. **Fig. 9. (a)** shows the variation of Nu_{av} with rotational, it observed that the maximum value of Nu_{av} occurs at the range of Ω from 0 to 0.7 and then decreasing with increasing of rotational speed Ω. This is because of reducing the effect of the secondary flow on the heat transfer process with increasing of Ω. **Fig. 9. (b)** indicates the increasing of θ_{av} with increasing of Ω. **Fig. 10. (a)** shows the variation of Nu_{av} with Re. It can be seen that the value of Nu_{av} is increased with increasing of Re number due to dominate of forced convection. **Fig. 10. (b)** show the effect of Re on θ_{av}. The increment of Re number gives increment in the mass and final decreasing in the temperature.

6. Conclusions :

The governing equations (mass, momentum and energy) for the steady, laminar, two dimensional, incompressible flow with Boussinseq approximation and constant fluid properties for rotating single cylinder in vented square enclosure are solved numerically using finite element method with FlexPDE soft package. The main conclusions:

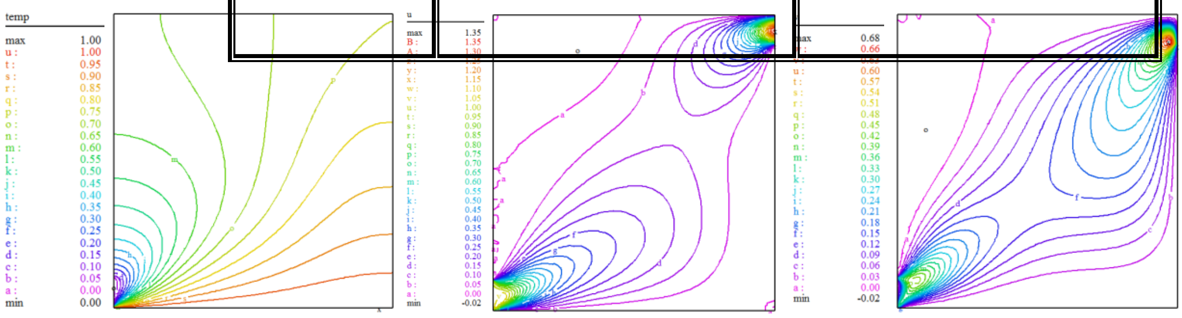
1-The average Nusselt number is increased with increasing Re, Ri and R and decreasing with increasing of the rotational velocity Ω.

2-When the angular rotating velocity equal to zero the behavior becomes of natural convection only.

3- The average temperature increases with Ri , R and Ω and decreases with Re.

Table -1. Comparison of the average temperature at different value of Ri with Abood et al. [10] at Pr = 0.71, Re = 100 and without rotation .

Ri	$\theta_{av.}$ present	$\theta_{av.}$ Ref.[10]
0	0.528	0.527
2	0.584	0.585
6	0.601	0.601
10	0.612	0.612
12	0.615	0.616



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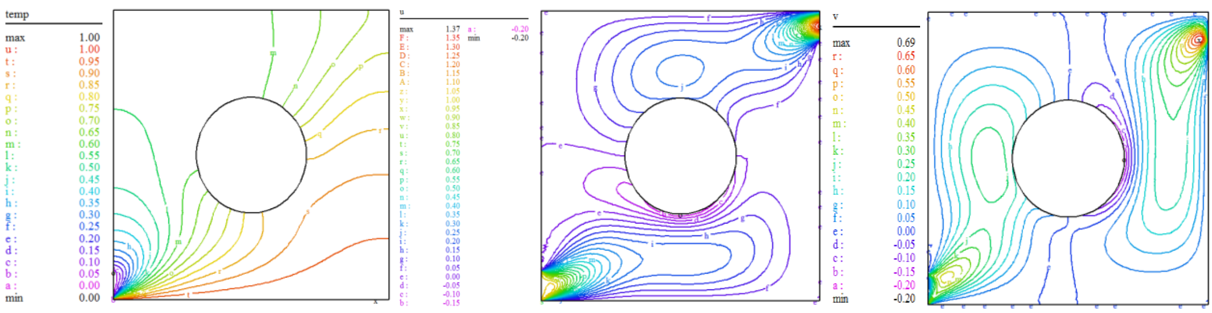

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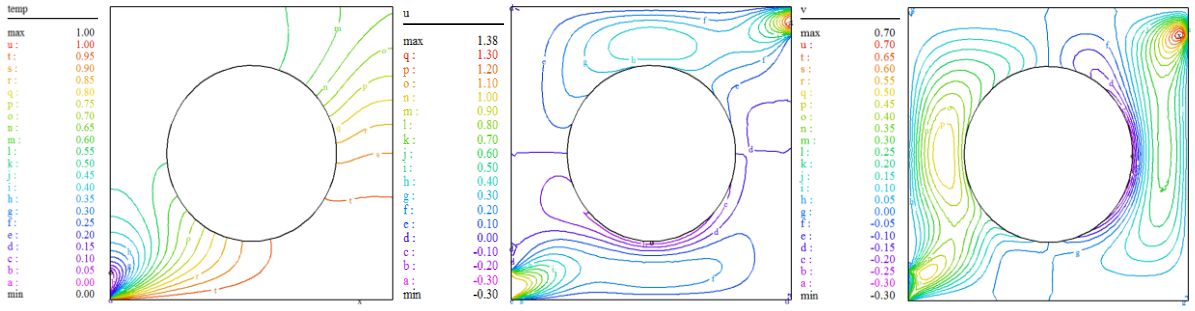
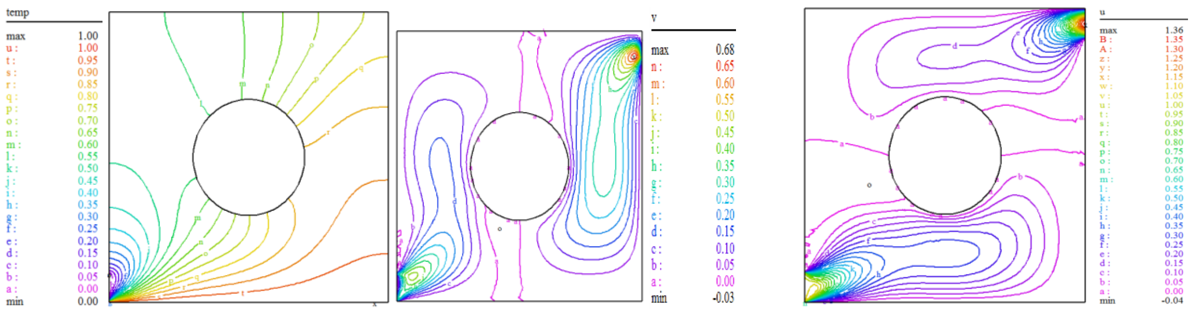


(c)

Fig.3. V-velocity (left), U-velocity (middle) and isotherms (right) for Ri=1, Re=50 and Ω=1 at (a) R=0 (b) R= 0.2 (c) R=0.3.



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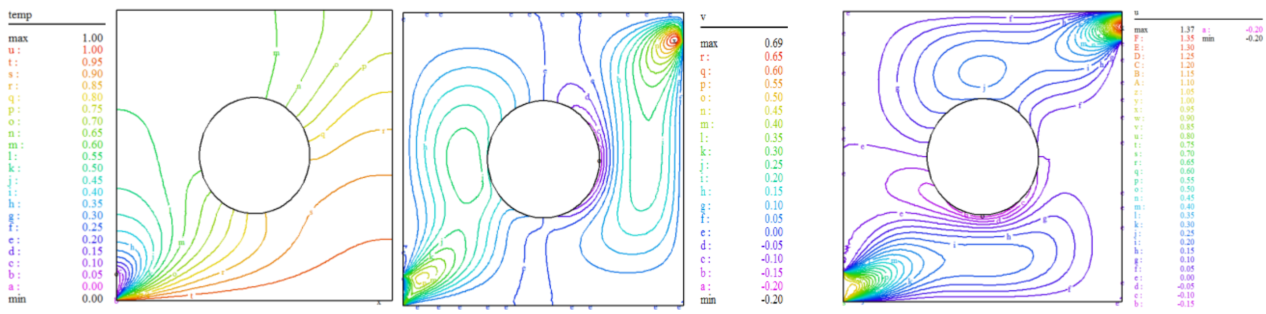

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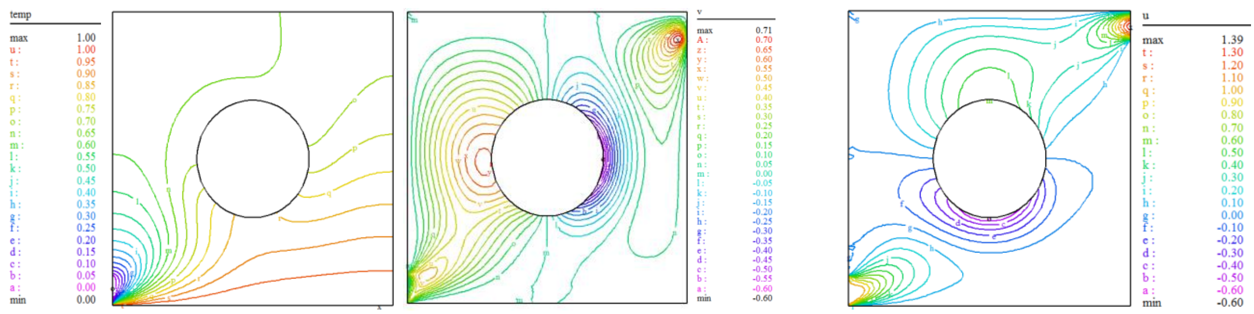
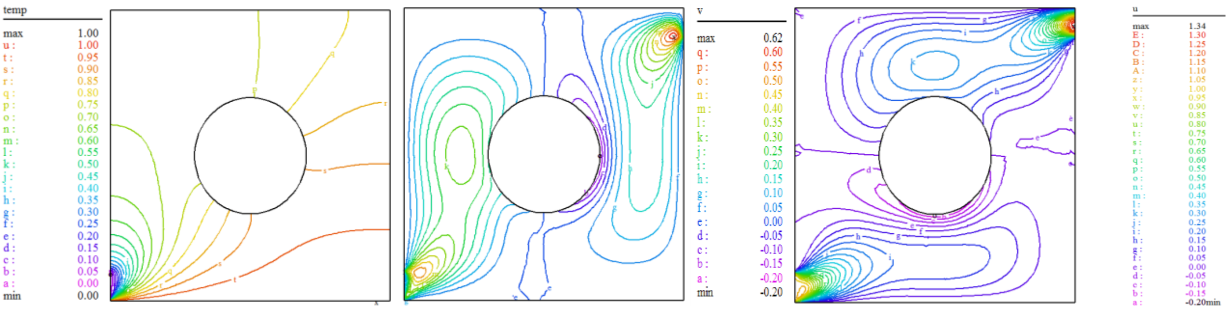


(c) Fig.4. V-velocity (left), U-velocity (middle) and isotherms (right) for Ri=1, Re=50 and R=0.2 at (a) Ω=0 (b) Ω=1 (c) Ω=3.



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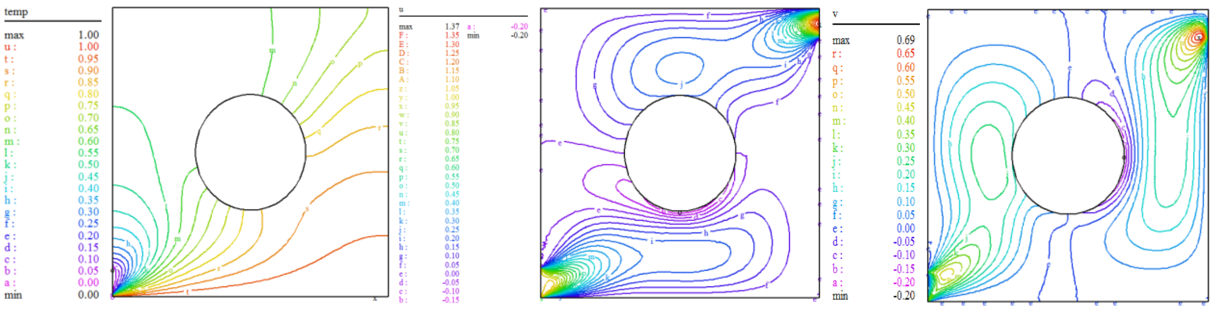

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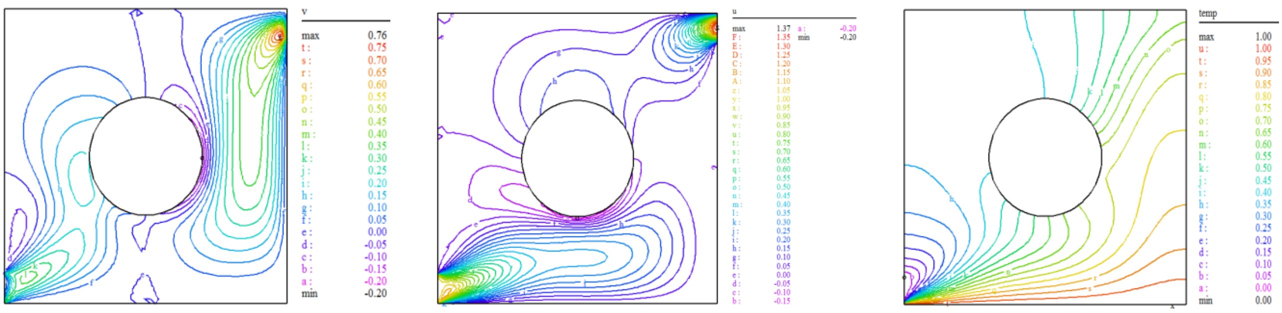
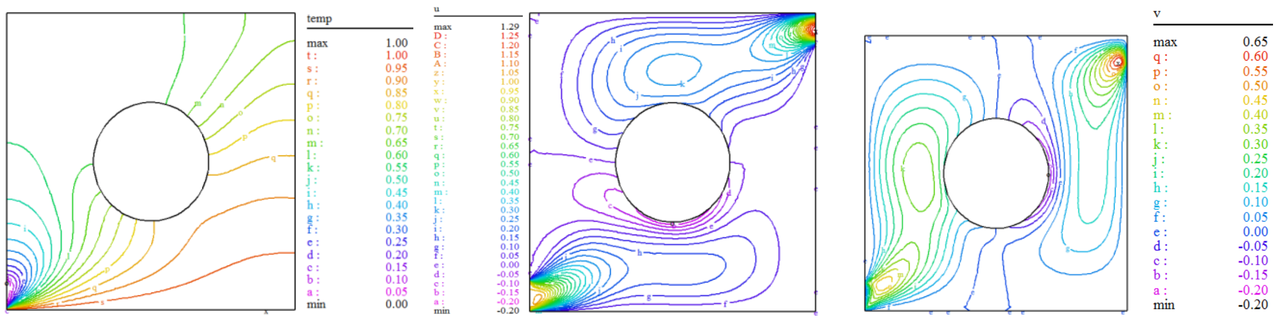


(c)

Fig.5. V-velocity (left), U-velocity (middle) and isotherms (right) for Ri=1, Ω=1 and R=0.2 at (a) Re=20 (b) Re=50 (c) Re=100.



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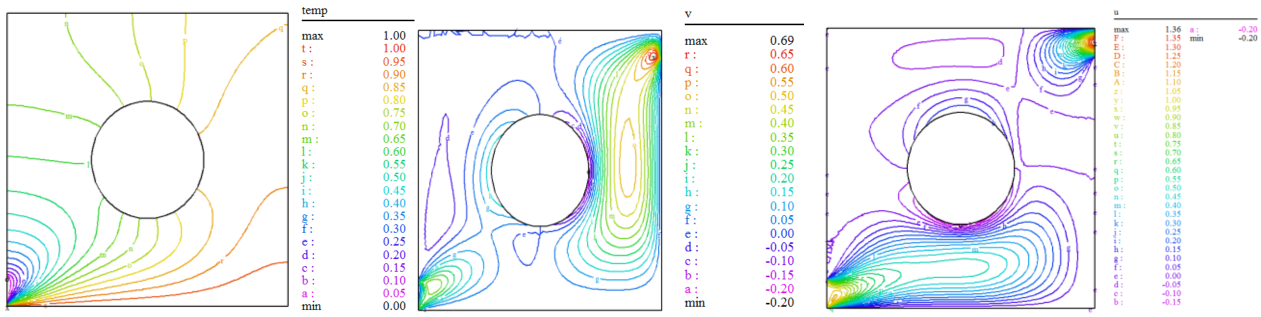

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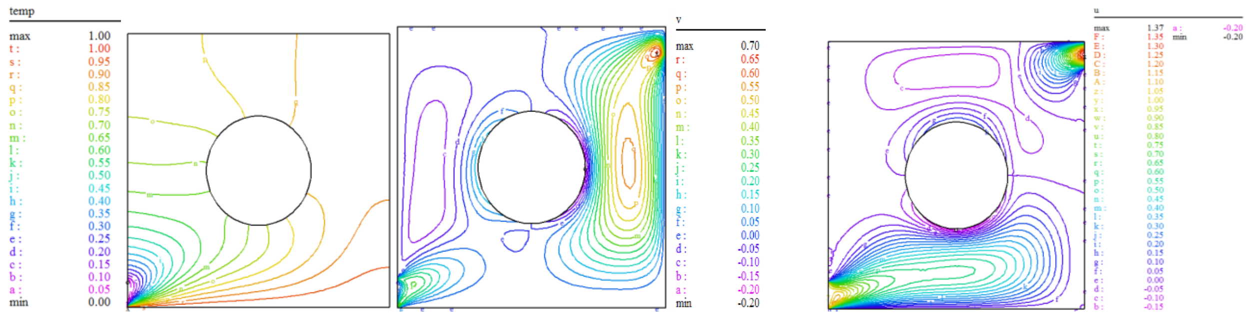
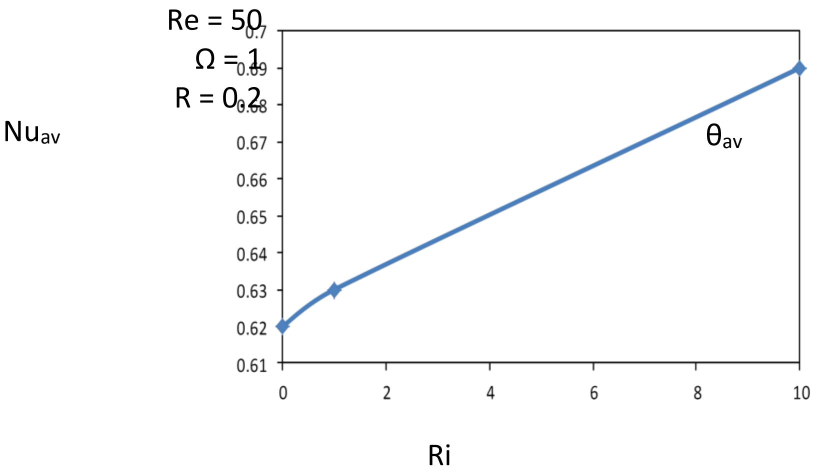
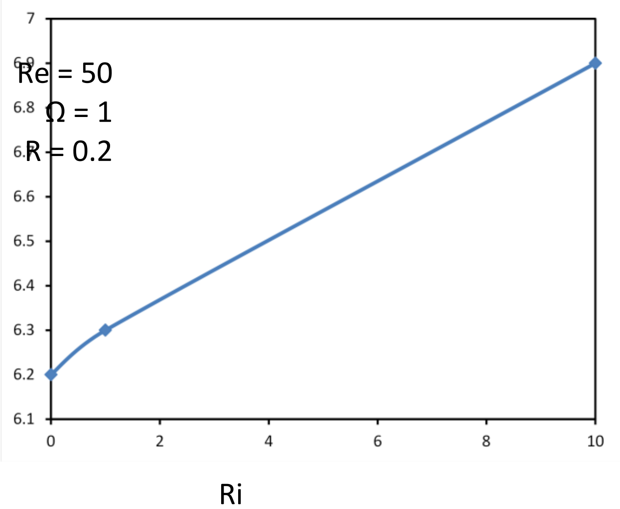


(c)

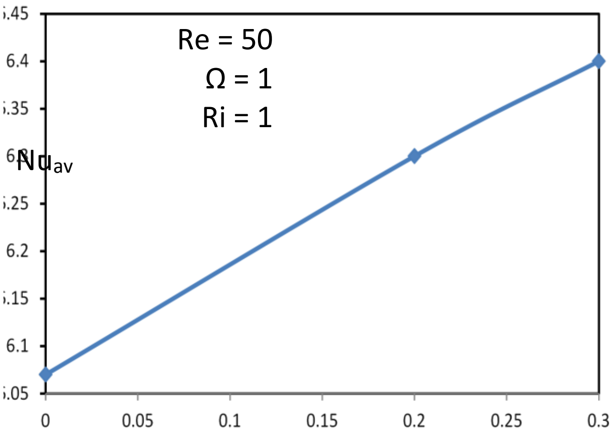
Fig.6. V-velocity (left), U-velocity (middle) and isotherms (right) for Re=50,Ω=1 and R=0.2 at (a) Ri=0 (b) Ri=6 (c) Ri=10.

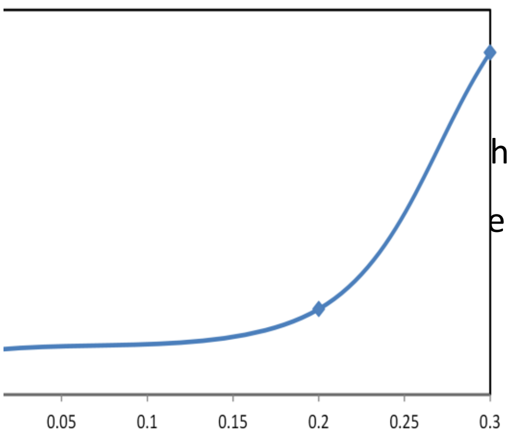




(a) (b)

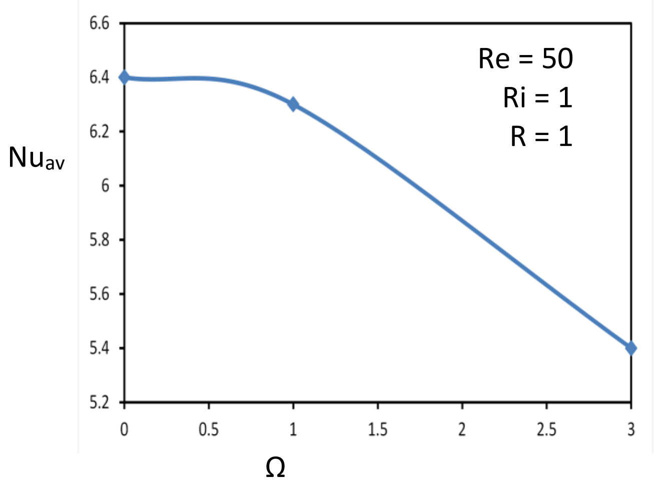
Fig. 7. (a) Variation of the Nusselt number with Richardson number (b) Variation of the average dimensionless temperature with Richardson number.





(b)

he Nusselt number with radius cylinder (b) e dimensionless temperature with radius of cylinder.



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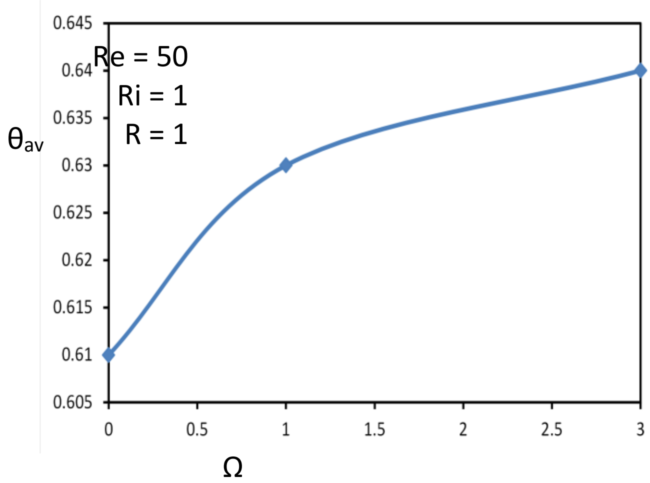
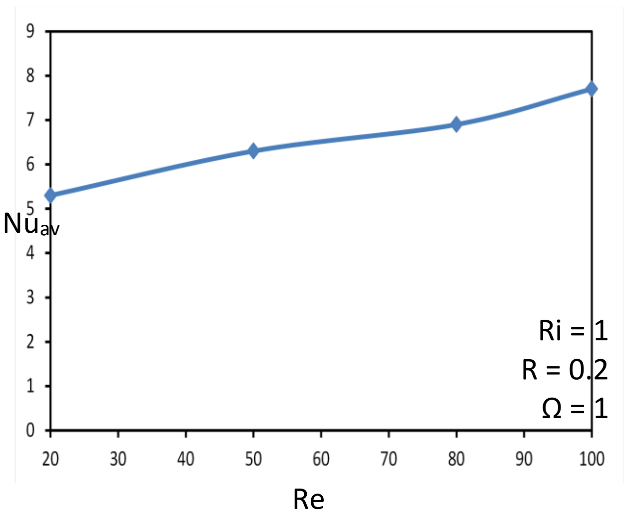


(b)

Fig. 9. (a) Variation of the Nusselt number with rotational velocity (b) Variation of the average dimensionless temperature with rotational velocity.



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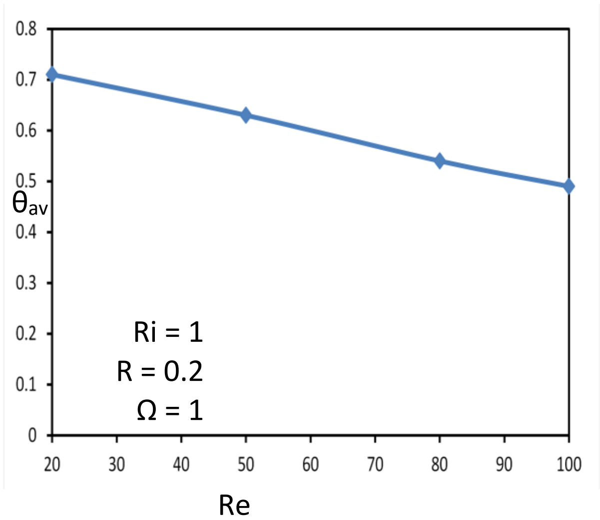

(b)

Fig. 10. (a) Variation of the Nusselt number with Reynolds number (b) Variation of the average dimensionless temperature with Reynolds number.

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8. Nomenclature

D	Diameter of rotating cylinder, m
g	Acceleration due to gravity, m/s^2
H	Height of square enclosure , m
k	Thermal conductivity , $W/m^2.K$
L	Length of the heated wall , m
N	Non-dimensional normal distance
Nu	Nusselt number
Nu_{av}	Average Nusselt number
P^*	Dimensionless pressure, $p/\rho u_{in}^2$
Pr	Prandtl number, υ/α
R	Dimensionless radius of the rotating cylinder, r/L
Ri	Richardson number, Gr/Re^2
Re	Reynolds number, $u_{in} L/\upsilon$
T	Temperature of the fluid in the enclosure, K
U,V	Non-dimensional velocity components , u/u_{in} , v/u_{in}
w	Height of inlet opening, 0.1 L
W	Non-dimensional height of inlet opening, w/L
X,Y	Non-dimensional coordinates, X=x/L, Y=y/L
Greek Symbols	
α	Thermal diffusivity of fluid, m^2/s
β	Thermal expansion coefficient, K^{-1}
θ	Dimensionless temperature, $\theta=(T-T_{in})/(T_h-T_{in})$

υ	Kinematics viscosity, m^2/s
ρ	Density , kg/m^3
ω	Angular rotational velocity of solid cylinder , rad/s
Ω	dimensionless angular rotational velocity, $\omega R/u_{in}$
Subscripts	
h	Hot
in	Inlet