System Response Analysis of Linear Time Invariant Multistage Feedback Amplifier Network

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Abstract

The analysis of voltage series multistage feedback amplifier network is achieved by evaluating the nodal admittance matrix of the equivalent circuit representing the replacing of each transistor in a stage by its high frequency small-signal model and the performing of the high order voltage transfer function of the system. The main reasons of treating such a wide-band amplifier network are of its stabilized voltage gain and its ability to amplify the pulses occurring in a television signal. The frequency response of the system is calculated and confirmed. System dynamics and variation of input signal are obtained by calculating the response of a continuous time system. The discrete-time equivalent to the analogue system allows the system designer to choose an appropriate pulse transfer function to investigate the performance of the system suitable for a given specifications and requirements. The software powerful MAT-LAB version 7.2 techniques is used for treating the single expression transfer function obtained by assigning numerical values and the response of the system in time and frequency domain.

تحليل استجابةِ نظامِ شبكةِ مضخّمِ تعليقاتِ الوقتِ الخطيِّ المتعددة المراحلِ الثابتة

المستخلص

يتناول البحث تحليل شبكة مضخم التغذية العكسية متعدد المراحل عن طريق حساب مصفوفة الدخل العقدية للدائرة المكافئة للمضخم المتمثلة بتبديل كل ترانزستور في المرحلة التي تمثله بنموذج التذبذب العالي صغير الإشارة وانجاز وظيفة نقل فولتية الطلب العالية للنظام.والسبب الرئيسي لمعالجة شبكة المضخم ذات النطاق الترددي العريض هو فولتية الكسب المستقرة لهذا المضخم وقابليته على تكبير النبضات التي تظهر في الإشارات التلفازية.أن الاستجابة الترددية للنظام ثم حسابها وتأكيدها وقد أمكن الحصول على ديناميكية النظام وتغيرات إشارة الإدخال وبحساب استجابة النظام للوقت المستمر.أن الوقت المفصل للنظام المكافئ للنظام التماثلي يسمح لمصمم النظام باختيار وظيفة نقل نبضي لبحث أداء النظام للتلاؤم مع أي مواصفات أو متطلبات معينة. وقد تم استخدام تقنية البرامج الفعالة MAT-LAB النسخة 7.2 لوظيفة النقل الأحادية المقطع المستحلة عن طريق تخصيص قيم عددية لاستجابة النظام في مديات الزمن و التردد.

1. Introduction

In a network design problems, numerical values are assigned to network elements and the simulation process resulted in a single expression transfer function to give the designer an ability to study the effect of network parameters on the network function in addition to the repeated usage of the information used.

The analysis of complex systems or networks can be facilitated through some insight into the internal structure of the elements constituting the system.

The wide-band voltage amplifier considered in the analysis of a multistage feedback amplifier network has an improved bandwidth through the usage of an additional stage and a flat response for minimum distortion in the signal. The negative feedback used in this network stabilizes the closed-loop gain of the amplifier at a lower and predictable gain level [1]. The voltage-series multistage feedback amplifier [2], depicted in Figure 1, is used in the analysis. The treatment of the network is based on the investigation of the behavior of the network under fixed parameters values. The advantages of treating such a network lie in the ability of controlling the gain by introducing a feedback through resistor combinations and the use of the feedback makes the system response insensitive to external disturbances and internal variation in the system parameters. The analysis is considered in the high frequency region and the flat response of the low pass filter network is obtained.

The descriptive and procedural side with the aid of additional facilities for examining the network gives a general and accurate circuit suitable for extension, modification and modular organization to the designer for future developments.

Complicated and tedious features of the network can be minimized by replacing each transistor in a single amplifier stage by its small-signal hybrid-π model valid in any topology as indicated in Figure (2), where the resistor R is

$$R = R_{c1} \| R_5 \| R_6$$

The nodal admittance matrix that describes the network is given by

$$\left[\frac{1}{r_{bb'}} + \frac{1}{r_{be'}} + s(C_e + C_c)\right]V_1 - \left(\frac{1}{r_{be'}} + sC_e\right)V_3 - sC_cV_2 = V_s / r_{bb'} \tag{1}$$

$$\left(\frac{1}{r_{bb'}} + \frac{1}{R} + sC_c\right)V_2 - sC_cV_1 - \frac{1}{r_{bb'}}V_4 = -g_m(V_1 - V_3) \tag{2}$$

$$\left(\frac{1}{r_{be'}}+\frac{1}{R_1}+\frac{1}{R_2}+sC_e\right)V_3-\left(\frac{1}{r_{be'}}+sC_e\right)V_1-\frac{1}{R_2}V_o=g_m\left(V_1-V_3\right) \qquad (3)$$

$$\left[\frac{1}{r_{bb'}}+\frac{1}{r_{be'}}+s\left(C_e+C_c\right)\right]V_4-\frac{1}{r_{bb'}}V_2-sC_cV_o=0 \qquad (4)$$

$$\left(\frac{1}{R_{c2}}+\frac{1}{R_2}+sC_c\right)V_o-\frac{1}{R_2}V_3-sC_cV_4=-g_mV_4 \qquad (5)$$

To characterize the input-output relationships of the LTI (linear time invariant) system described by the nodal admittance matrix above the transfer function can be found as

$$\frac{V_o}{V_s}=\frac{N}{D}=\frac{\begin{vmatrix} \frac{1}{r_{bb'}}+\frac{1}{r_{be'}}+s\left(C_e+C_c\right) & -sC_c & -\frac{1}{r_{be'}}-sC_e & 0 & \frac{1}{r_{bb'}} \\ g_m-sC_c & \frac{1}{r_{bb'}}+\frac{1}{R}+sC_c & -g_m & -\frac{1}{r_{bb'}} & 0 \\ -g_m-\frac{1}{r_{be'}}-sC_e & 0 & g_m+\frac{1}{r_{be'}}+\frac{1}{R_1}+\frac{1}{R_2}+sC_e & 0 & 0 \\ 0 & -\frac{1}{r_{bb'}} & 0 & \frac{1}{r_{bb'}}+\frac{1}{r_{be'}}+s\left(C_e+C_c\right) & 0 \\ 0 & 0 & -\frac{1}{R_2} & g_m-sC_c & 0 \end{vmatrix}}{\begin{vmatrix} \frac{1}{r_{bb'}}+\frac{1}{r_{be'}}+s\left(C_e+C_c\right) & -sC_c & -\frac{1}{r_{be'}}-sC_e & 0 & 0 \\ g_m-sC_c & \frac{1}{r_{bb'}}+\frac{1}{R}+sC_c & -g_m & -\frac{1}{r_{bb'}} & 0 \\ -g_m-\frac{1}{r_{be'}}-sC_e & 0 & g_m+\frac{1}{r_{be'}}+\frac{1}{R_1}+\frac{1}{R_2}+sC_e & 0 & -\frac{1}{R_2} \\ 0 & -\frac{1}{r_{be'}} & 0 & \frac{1}{r_{bb'}}+\frac{1}{r_{be'}}+s\left(C_e+C_c\right) & -sC_c \\ 0 & 0 & -\frac{1}{R_2} & g_m-sC_c & -\frac{1}{R_{c1}}+\frac{1}{R_2}sC_c \end{vmatrix}}$$

where (6)

$$N = (1/r_{bb'})[(-1/r_{bb'})(g_m - sC_c)\{(-g_m)(-g_m - 1/r_{b'e} - sC_e) - (g_m - sC_c)(g_m + 1/r_{b'e} + 1/R_1 + 1/R_2 + sC_e\} + (-g_m - 1/r_{b'e} - sC_e)(-1/R_2)\{(1/r_{bb'} + 1/R + sC_c)(1/r_{bb'} + 1/r_{b'e} + s(C_e + C_c)) - (-1/r_{bb'})(-1/r_{bb'})\}] \tag{7}$$

and

$$D = (1/r_{bb'} + 1/r_{b'e} + s(C_e + C_c))[(1/r_{bb'} + 1/R + sC_c)(g_m + 1/r_{b'e} + 1/R_1 + 1/R_2 + sC_e)(1/r_{bb'} + 1/r_{b'e} + s(C_e + C_c))(1/R_{c1} + 1/R_2 + sC_c) - (-sC_c)(g_m - sC_c)\} + (-1/R_2)(-1/R_2)\{(-1/r_{bb'})(-1/r_{bb'}) - (1/r_{bb'} + 1/R + sC_c)(1/r_{bb'} + 1/r_{b'e} + s(C_e + C_c)\} - (-1/r_{bb'})\{(-g_m)(-1/R_2)(g_m - sC_c) + (-1/r_{bb'})(g_m + 1/r_{b'e} + 1/R_1 + 1/R_2 + sC_e)(1/R_{c1} + 1/R_2 + sC_c)\}] + (-sC_c)[(g_m - sC_c)(g_m + 1/r_{b'e} + 1/R_1 + 1/R_2 + sC_e)\{(-sC_c)(g_m - sC_c) - ((1/r_{bb'} + 1/r_{b'e} + s(C_e + C_c))(1/R_{c1}$$

$$+ 1/R_2 + sC_c)\} + (-g_m)(-g_m - 1/r_{b'e} - sC_e)\{((1/r_{bb'} + 1/r_{b'e} + s(C_e + C_c))(1/R_{c1} + 1/R_2 + sC_c) - (-sC_c)(g_m - sC_c)\} + (-1/R_2)\{(g_m - sC_c)(-1/R_2)(1/r_{bb'} + 1/r_{b'e} + s(C_e + C_c)) + (-1/r_{bb'})(-g_m - 1/r_{b'e} - sC_e)(-sC_c)\}] + (-1/r_{b'e} - sC_e)[(1/r_{bb'} + 1/R + sC_c)(-g_m - 1/r_{b'e} - sC_e)\{(-sC_c)(g_m - sC_c) - (1/r_{bb'} + 1/r_{b'e} + s(C_e + C_c))(1/R_{c1} + 1/R_2 + sC_c)\} + (-1/r_{bb'})\{(g_m - sC_c)(-1/R_2)(g_m - sC_c) + (-1/r_{bb'})(-g_m - 1/r_{b'e} - sC_e)(1/R_{c1} + 1/R_2 + sC_c)\}] \tag{8}$$

For hybrid-π parameters value, the magnitudes used for the elements of the hybrid-π model at room temperature are;

$g_m = 50\,\mathrm{mA/V}$, $r_{bb'} = 100\,\Omega$, $r_{b'e} = 1\,\mathrm{K\Omega}$, $C_e = 4\,\mathrm{pF}$, $C_c = 98\,\mathrm{pF}$

where g_m is the transistor transconductance, $r_{bb'}$ is the ohmic base spreading resistance, $r_{b'e}$ is the input resistance, C_e is the sum of the emitter diffusion capacitance and the emitter junction capacitance, and C_c is the collector junction capacitance. The magnitudes for the other elements of the network are;

$R_1 = 140\,\mathrm{K\Omega}$, $R_2 = 50\,\mathrm{K\Omega}$, $R_{c1} = 12\,\mathrm{K\Omega}$, $R_{e1} = 5\,\mathrm{K\Omega}$, $R_3 = 49\,\mathrm{K\Omega}$, $R_4 = 32\,\mathrm{K\Omega}$,

$R_{c2} = 4.5\,\mathrm{K\Omega}$, $R_{e2} = 5\,\mathrm{K\Omega}$, $R_1 = 125\,\Omega$, $R_2 = 5.2\,\mathrm{K\Omega}$, $C_1 = 6\,\mu\mathrm{F}$, $C_2 = 6\,\mu\mathrm{F}$,

$C_3 = 10\,\mu\mathrm{F}$, $C_4 = 48\,\mu\mathrm{F}$, $C_5 = 48\,\mu\mathrm{F}$, $C_6 = 6\,\mu\mathrm{F}$.

2. Transfer function analysis

According to the above magnitudes, the transfer function $TF_{A(s)}$ of the continuous-time multistage feedback amplifier network is given by

$$TF_A(s) = 5.3686\times10^7 \frac{\left(s-7.4716\times10^9\right)\left(s+1.3852\times10^9\right)\left(s-0.8553\times10^9\right)}{\left(s+1.9874\times10^9\right)\left(s+0.6597\times10^9\right)\left(s+0.2101\times10^9\right)\left(s+0.0425\times10^9\right)} \qquad (9)$$

where s is a complex variable.

The partial fraction expansion [3] of $TF_{A(s)}$ has the form

$$TF_A(s) = \frac{0.1894\times10^9}{s+1.9874\times10^9} + \frac{1.3023\times10^9}{s+0.6597\times10^9} - \frac{3.8542\times10^9}{s+0.2101\times10^9} + \frac{2.4162\times10^9}{s+0.0425\times10^9} \qquad (10)$$

Clearly, the amplifier has three zeros

$z_1 = 7.4716\times10^9 \qquad z_2 = -1.3852\times10^9 \qquad z_3 = 0.8553\times10^9$

and four poles

$p_1 = -1.9874\times10^9 \qquad p_2 = -0.6597\times10^9$

$p_3 = -0.2101\times10^9 \qquad p_4 = -0.0425\times10^9$

The zeros and poles lie in the high frequency band. The frequency response characteristics of the amplifier are obtained by the Bode plot. The adjusting of these characteristics by using several design criteria gives acceptable transient-response characteristics [4], the high frequency response can also be improved by using current amplifier [5]. Substituting $s = j\omega$ in Eq. 9 gives

$$TF_A(j\omega) = \frac{40.5937\left(1-j\frac{\omega}{7.4716\times10^9}\right)\left(1+j\frac{\omega}{1.3852\times10^9}\right)\left(1-j\frac{\omega}{0.8553\times10^9}\right)}{\left(1+j\frac{\omega}{1.9874\times10^9}\right)\left(1+j\frac{\omega}{0.6597\times10^9}\right)\left(1+j\frac{\omega}{0.2101\times10^9}\right)\left(1+j\frac{\omega}{0.0425\times10^9}\right)} \qquad (11)$$

where ω is the angular frequency. With the aid of MATLAB numeric computation software [6], the Bode magnitude and phase plot are depicted in Figs. 3 and 4. It can be seen that the peak gain is $32.2\,\text{dB}$ at $\omega = 0.0848 \times 10^6$ rad/sec $(f = 13.5\,\text{KHz})$, where f is the frequency. The $-3\,\text{dB}\,(28.84\,\text{or}\,29.2\,\text{dB})$ occurs at $\omega = 4.05 \times 10^7$ rad/sec $(f = 6.455\,\text{MHz})$.

3. Confirmation

A calculations can be made to confirm the value of the peak gain obtained from the Bode plot with that evaluated above in the following procedure;

1- Evaluation of the output resistances and the voltage gain of each transistor.
2- Obtaining the voltage gain of the amplifier without feedback.
3- Determination of the feedback network.
4- Calculation of the voltage gain of the amplifier with feedback.

According to the stated procedure,

$$R_{L1} = R_{c1} \| R_3 \| R_4 \| h_{ie2} = 12\,\text{K}\Omega \| 49\,\text{K}\Omega \| 32\,\text{K}\Omega \| 1.1\,\text{K}\Omega = 957\,\Omega$$

$$R_{L2} = R_{c2} \| (R_1 + R_2) = 4.5\,\text{K}\Omega \| (125\,\Omega + 5.2\,\text{K}\Omega) = 2.438\,\text{K}\Omega$$

where R_{L1}, R_{L2}, h_{ie2} are the output resistance of Q_1, the output resistance of Q_2, the input resistance with output short-circuited hybrid parameter of Q_2, respectively. The values of the voltage gain of Q_1 and Q_2 are A_{v1} and A_{v2} are given by, (neglecting the reverse –open-circuit voltage amplification hybrid parameter h_{re} and the output conductance with input open-circuited hybrid parameter h_{oe}), so

$$A_{v1} = \frac{-h_{fe1} R_{L1}}{h_{ie1} + (1 + h_{fe1})(R_1 \| R_2)} = \frac{-50(957\,\Omega)}{1.1\,\text{K}\Omega + (51)(125\,\Omega \| 5.2\,\text{K}\Omega)} = -6.427$$

$$A_{v2} = \frac{-h_{fe2} R_{L2}}{h_{ie2}} = \frac{-50(2.438\,\text{K}\Omega)}{1.1\,\text{K}\Omega} = -110.81$$

$$A_{vt} = A_{v1} \times A_{v2} = -6.427 \times -110.81 = 712.17$$

where h_{fe1} and h_{fe2} are the short-circuit current gain hybrid parameters of Q_1 and Q_2, respectively, h_{ie1} is the input resistance hybrid parameter of Q_1, and A_{vt} is the voltage gain of the amplifier without feedback. The value of the feedback network, β, is given by

$$\beta = R_1 / (R_1 + R_2) = 125\,\Omega / (125\,\Omega + 5.2\,\text{K}\Omega) = 0.0234$$

The voltage gain of the amplifier with feedback is

$$A_{vf} = \frac{A_{vt}}{1+\beta A_{vt}} = \frac{712.17}{1+(0.0234)(712.17)} = 40.326 = 32.11\,\text{dB}$$

since $|A_{vf}| < |A_{vt}|$ the feedback is negative. The gain can also be confirmed from Eq. 9 as

$$\frac{5.3638\times10^{7} z_1 z_2 z_3}{p_1 p_2 p_3 p_4} = \frac{5.3638\times10^{7}\times7.4716\times10^{9}\times1.3852\times10^{9}\times0.8553\times10^{9}}{1.9874\times10^{9}\times0.6597\times10^{9}\times0.2101\times10^{9}\times0.0425\times10^{9}}$$

$$= 40.59 = 32.16\,\text{dB}.$$

4. Continuous time responses

Complete information about dynamic characteristics of the system can be obtained by exciting it with an impulse input and measuring the output. The transfer function stated previously; however, contain same information about system dynamics as the response of a linear system to a unit-impulse input when the initial conditions are zero. The unit-impulse response is obtained by taking the inverse Laplace transform to Eq. 10.

$$\begin{aligned} h_A(t) &= 0.1894\times10^{9}\exp\left(-1.9874\times10^{9}t\right) + 1.3023\times10^{9}\exp\left(-0.6597\times10^{9}t\right) \\ &\quad - 3.8542\times10^{9}\exp\left(-0.2101\times10^{9}t\right) + 2.4162\times10^{9}\exp\left(-0.0425\times10^{9}t\right) \end{aligned} \tag{12}$$

where $h_A(t)$ is a function of time t.

The amplifier at high frequencies responds to rapid variations in signals, the step input is of the most available sudden function applied to the amplifier. The input-output relationship of the amplifier $TF_{st-A}(s)$ subjected to a step input is given by

$$TF_{st-A}(s) = \frac{1}{s}\left(\frac{0.1894\times10^{9}}{s+1.9874\times10^{9}} + \frac{1.3023\times10^{9}}{s+0.6597\times10^{9}} - \frac{3.8542\times10^{9}}{s+0.2101\times10^{9}} + \frac{2.4162\times10^{9}}{s+0.0425\times10^{9}}\right) \tag{13}$$

$$= \frac{0.0953}{s} - \frac{0.0953}{s + 1.9874 \times 10^9} + \frac{1.974}{s} - \frac{1.974}{s + 0.6597 \times 10^9} - \frac{18.3445}{s} + \frac{18.3445}{s + 0.2101 \times 10^9} + \frac{56.8517}{s} - \frac{56.8517}{s + 0.0425 \times 10^9} \quad (14)$$

Taking the inverse Laplace transform yields the unit step response of the system given by

$$h_{st-A}(t) = 0.0953\left[1 - \exp\left(-1.9874 \times 10^9 t\right)\right] + 1.974\left[1 - \exp\left(-0.6597 \times 10^9 t\right)\right] - 18.3445\left[1 - \exp\left(-0.2101 \times 10^9 t\right)\right] + 56.8517\left[1 - \exp\left(-0.0425 \times 10^9 t\right)\right] \quad (15)$$

For gradual changing function, the unit ramp function is a good test signal. The input-output relationship of the amplifier $TF_{ra-A}(s)$ subjected to a unit ramp input is given by

$$TF_{ra-A}(s) = \frac{1}{s^2}\left(\frac{0.1894 \times 10^9}{s + 1.9874 \times 10^9} + \frac{1.3023 \times 10^9}{s + 0.6597 \times 10^9} - \frac{3.8542 \times 10^9}{s + 0.2101 \times 10^9} + \frac{2.4162 \times 10^9}{s + 0.0425 \times 10^9}\right) \quad (16)$$

$$= -\frac{0.0479 \times 10^9}{s} + \frac{0.0953}{s^2} - \frac{0.0479 \times 10^9}{s + 1.9874 \times 10^9} + \frac{2.9923 \times 10^9}{s} + \frac{1.974}{s^2} - \frac{2.9923 \times 10^9}{s + 0.6597 \times 10^9} + \frac{87.3136 \times 10^9}{s} - \frac{18.3445}{s^2} - \frac{87.3136 \times 10^9}{s + 0.2101 \times 10^9} - \frac{1337.6885 \times 10^9}{s} + \frac{56.8517}{s^2} - \frac{1337.6885 \times 10^7}{s + 0.0425 \times 10^9} \quad (17)$$

Taking the inverse Laplace transform yields the unit ramp response of the system as a function of time given by

$$h_{ra-A}(t) = 0.0953t - 0.0479 \times 10^9\left[1 - \exp\left(-1.9874 \times 10^9 t\right)\right] + 1.974t - 2.9923 \times 10^9 \left[1 - \exp\left(-0.6597 \times 10^9 t\right)\right] - 18.3445t + 87.3136 \times 10^9\left[1 - \exp\left(-0.2101 \times 10^9 t\right)\right] + 56.8517t - 1337.6885 \times 10^9\left[1 - \exp\left(-0.0425 \times 10^9 t\right)\right] \quad (18)$$

The unit impulse, step and ramp responses of the multistage feedback amplifier are depicted in Figs. 5, 6 and 7, respectively. The type of transient response is determined by the closed-loop poles, while the shape of the transient response is primarily described by the closed-loop zeros. The poles of the high order transfer function of the multistage feedback amplifier does not possess complex-conjugate values so the system is non oscillatory. It can be seen also from Figs. 5 and 6 that the system is stable since the magnitudes of the poles lie in the left-half s-plane [7] and the response will die out because the exponential terms in Eqs. 12 and 15 will approaches zero as time increases.

5. Discrete-time system

To simulate the continuous-time amplifier, the impulse invariant method is used [8]. So the design of the discrete-time amplifier from the continuous-time amplifier is as follows; the z-transform of the unit sample response system, $TF_D(z)$ is given by

$$TF_D(z) = \frac{0.1894\times10^9}{1-e^{-1.9874\times10^9 T_s} z^{-1}} + \frac{1.3023\times10^9}{1-e^{-0.6597\times10^9 T_s} z^{-1}} - \frac{3.8542\times10^9}{1-e^{-0.2101\times10^9 T_s} z^{-1}} + \frac{2.4162\times10^9}{1-e^{-0.0425\times10^9 T_s} z^{-1}} \tag{19}$$

where $z = e^{sT_s}$ is a complex variable, T_s is the sampling period and $n = 0,1,2....$ is an integer. Clearly the discrete-time system described by Eq. 19 is stable [9] and $TF_D(z)$ approximates the system performance. The unit sample response of the analogue amplifier has a unit sample response equivalent to the unit sample response $h_D(n)$ obtained from Eq. 21 as

$$h_D(n) = 0.1894\times10^9 \exp\left(-1.9874\times10^9 T_s\right)^n + 1.3023\times10^9 \exp\left(-0.6597\times10^9 T_s\right)^n - 3.8542\times10^9 \exp\left(-0.2101\times10^9 T_s\right)^n + 2.4162\times10^9 \exp\left(-0.0425\times10^9 T_s\right)^n \tag{20}$$

The unit sample response system [10] for $T_s = 2\,\text{nS}$ and the parallel realization of Eq. 19 are shown in Figs. 8 and 9. It is interesting to compare the magnitude response of two amplifiers, for the analogue amplifier the magnitude response, $M_A(\omega)$ is

$$M_A(\omega)=\frac{0.1894\times10^9}{\sqrt{\omega^2+\left(1.9874\times10^9\right)^2}}+\frac{1.3023\times10^9}{\sqrt{\omega^2+\left(0.6597\times10^9\right)^2}}-\frac{3.8542\times10^9}{\sqrt{\omega^2+\left(0.2101\times10^9\right)^2}}+\frac{2.4162\times10^9}{\sqrt{\omega^2+\left(0.0425\times10^9\right)^2}} \qquad (21)$$

and for the discrete-time amplifier, the magnitude response $M_D(\omega)$ is

$$M_D(\omega)=\frac{0.1894\times10^9}{\sqrt{1-2e^{-1.9874\times10^9T_s}\cos(\omega T_s)+\left(e^{-1.9874\times10^9T_s}\right)^2}}+\frac{1.3023\times10^9}{\sqrt{1-2e^{-0.6597\times10^9T_s}\cos(\omega T_s)+\left(e^{-0.6597\times10^9T_s}\right)^2}}$$

$$-\frac{3.8542\times10^9}{\sqrt{1-2e^{-0.2101\times10^9T_s}\cos(\omega T_s)+\left(e^{-0.2101\times10^9T_s}\right)^2}}+\frac{2.4162\times10^9}{\sqrt{1-2e^{-0.0425\times10^9T_s}\cos(\omega T_s)+\left(e^{-0.0425\times10^9T_s}\right)^2}} \qquad (22)$$

The magnitude responses of the two amplifiers are depicted in Figs. 10 and 11. Due to the sampling operation, the amplitude response of the discrete-time system is scales by $1/T_s$. Therefore, a multiplication of $TF_D(z)$ by $1/T_s$ is required to approximate the amplitude response of the discrete-time system to the amplitude response of the analog system [11]. Thus the unit sample response, $TF_{iD}(z)$, of the impulse invariant discrete-time system equivalent to $h_A(t)$ is given by

$$TF_{iD}(z)=\frac{0.1894\times10^{9}T_s}{1-e^{-1.9874\times10^{9}T_s}z^{-1}}+\frac{1.3023\times10^{9}T_s}{1-e^{-0.6597\times10^{9}T_s}z^{-1}}-\frac{3.8542\times10^{9}T_s}{1-e^{-0.2101\times10^{9}T_s}z^{-1}}+\frac{2.4162\times10^{9}T_s}{1-e^{-0.0425\times10^{9}T_s}z^{-1}} \qquad (23)$$

The dc response for the analogue system is

$$TF_A(0)=\frac{0.1894\times10^{9}}{1.9874\times10^{9}}+\frac{1.3023\times10^{9}}{0.6597\times10^{9}}-\frac{3.8542\times10^{9}}{0.2101\times10^{9}}+\frac{2.4162\times10^{9}}{0.0425\times10^{9}}$$
$$=40.576=32.6155\,\text{dB}$$

For the discrete-time amplifier, since for any variable x , if the sampling frequency is high, T_s is small, and

$$\exp(-xT_s)=1-(-xT_s)=1+xT_s$$

with this approximation,

$$TF_D(0)=TF_A(0)$$

with good agreement. The input $X(z)$ and the output $Y(z)$ relationship of the discrete-time amplifier is given by

$$TF_D(z)=\frac{Y(z)}{X(z)}=\frac{5.37\times10^{7}-6.088\times10^{7}z^{-1}+4.5246\times10^{8}z^{-2}-4.5502\times10^{7}z^{-3}}{1-1.8615z^{-1}+1.0591z^{-2}-0.1805z^{-3}+0.003z^{-4}} \qquad (24)$$

and the corresponding difference equation is

$$y[n]-1.8615y[n-1]+1.0592y[n-2]-0.1805y[n-3]+0.003y[n-4]=5.37\times10^{7}x[n]$$
$$-6.088\times10^{7}\,\mathrm{x}[\mathrm{n}-1]+4.5246\times10^{8}x[n-2]-4.5502\times10^{7}x[n-3] \qquad (25)$$

Taking the Fourier transform of this equation gives

$$TF_D(j\omega)=\frac{Y(j\omega)}{X(j\omega)}$$
$$=\frac{5.37\times10^7-6.088\times10^7(j\omega)^{-1}+4.5246\times10^8(j\omega)^{-2}-4.5502\times10^7(j\omega)^{-3}}{1-1.8615(j\omega)^{-1}+1.0591(j\omega)^{-2}-0.1805(j\omega)^{-3}+0.003(j\omega)^{-4}} \quad (26)$$

The phase response of the discrete-time system can be determined as

$$\begin{aligned}&=\tan^{-1}\angle[5.37\times10^7\sin(4\omega)-6.088\times10^7\sin(3\omega)+4.5246\times10^8\sin(2\omega)-3.5502\times10^7\\&\sin(\omega)]/[5.37\times10^7\cos(4\omega)-6.088\times10^7\cos(3\omega)+4.5246\times10^8\cos(2\omega)-3.5502\times10^7\\&\cos(\omega)]-\tan^{-1}\angle[\sin(4\omega)-1.8615\sin(3\omega)+1.0591\sin(2\omega)-0.1805\sin(\omega)]/[\cos(4\omega)-\\&1.8615\cos(2\omega)+1.0591\cos(2\omega)-0.1805\cos(\omega)]\end{aligned} \quad (27)$$

From the observation of the frequency response of the discrete-time system and the frequency response of the analog system, it can be seen that the first is periodic function of ω while the last is aperiodic. The unit step and ramp sample responses for the discrete-time amplifier are given by

$$\begin{aligned}h_{st-D}(n)=0.0953&\left[(1)^n-\left(e^{-1.9874\times10^9T_s}\right)^n\right]+1.974\left[(1)^n-\left(e^{-0.6597\times10^9T_s}\right)^n\right]\\&-18.3445\left[(1)^n+\left(e^{-0.2101\times19^9T_s}\right)^n\right]+56.8517\left[(1)^n-\left(e^{-0.0425\times10^9T_s}\right)^n\right]\end{aligned} \quad (28)$$

and

$$\begin{aligned}h_{ra-D}(n)=0.0953n-0.0479\times10^9&\left[(1)^n-\left(e^{-1.9874\times10^9T_s}\right)^n\right]+1.974n-2.9923\times10^9\\&\left[(1)^n-\left(e^{-0.6597\times10^9T_s}\right)^n\right]-18.3445n+87.3136\times10^9\left[(1)^n-\left(e^{-0.2101\times19^9T_s}\right)^n\right]\\&+56.8517n-1337.6885\times10^9\left[(1)^n-\left(e^{-0.0425\times10^9T_s}\right)^n\right]\end{aligned} \quad (29)$$

and both are depicted in Figs. 12 and 13, respectively for $T_s=2\,\text{nS}$.

6. Conclusions

The analysis of a linear time invariant voltage series multistage feedback amplifier network at high frequencies was performed. The nodal admittance matrix of the five nodes network was constructed. An additional stage to the amplifier is possible to increase the voltage gain but this complicates the nodal admittance matrix. The transfer function resulted from the solution of the linear equations representing the system is the key function and it is useful for both design and synthesis technique by selecting numerical values in the simulation process. The value of the peak gain resulted from the nodal analysis and then evaluated from the transfer function's Bode plot was $32.2\,\mathrm{dB}\,(40.738)$ is confirmed with that calculated from the evaluating the voltage gain in each stage (with the feedback network) which was $32.11\,\mathrm{dB}\,(40.326)$ and the small difference between the two values occurs because of the simulation process in the MATLAB. The transfer function has three zeros and four poles due to the four capacitors in the network. The system exhibits high performance due to small time domain quantities and the transient response of the system was nonoscillatory, in addition to the system is stable since all the poles lies to the left of the imaginary axis. The impulse invariant method is used to simulate the amplifier and a same magnitude is obtained for both the continuous-time and discrete-time system. The steady state time which was 0.15×10^{-6} s was the same in continuous and discrete time system. The step and ramp response for the discrete-time system is analogous to the continuous-time system.

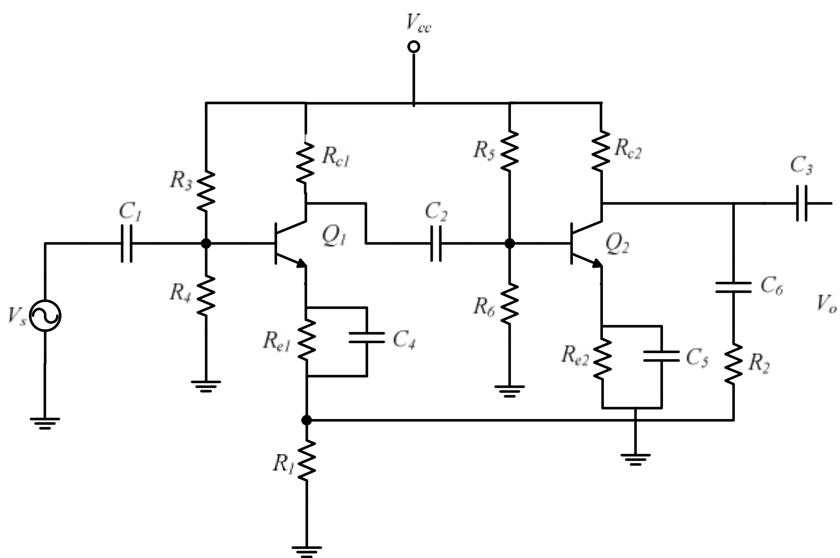

Figure (1). Multistage feedback amplifier network.

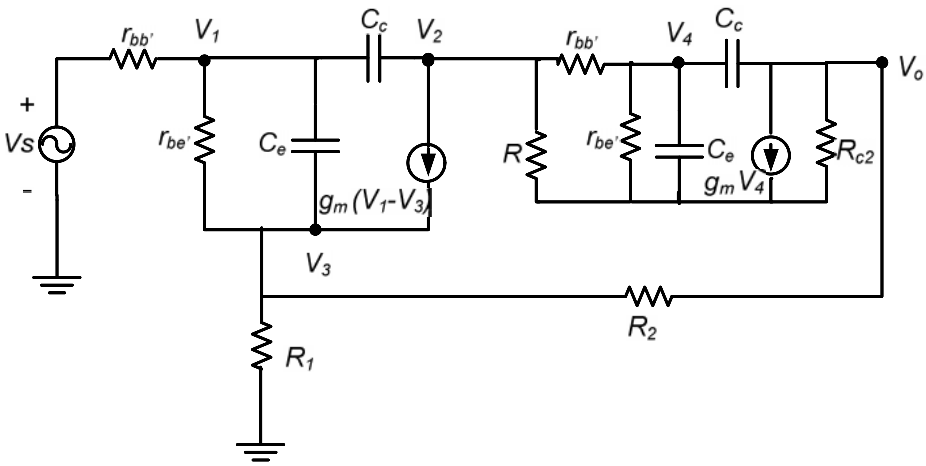

Figure (2). Small-signal hybrid-π model for voltage-series multistage feedback amplifier.

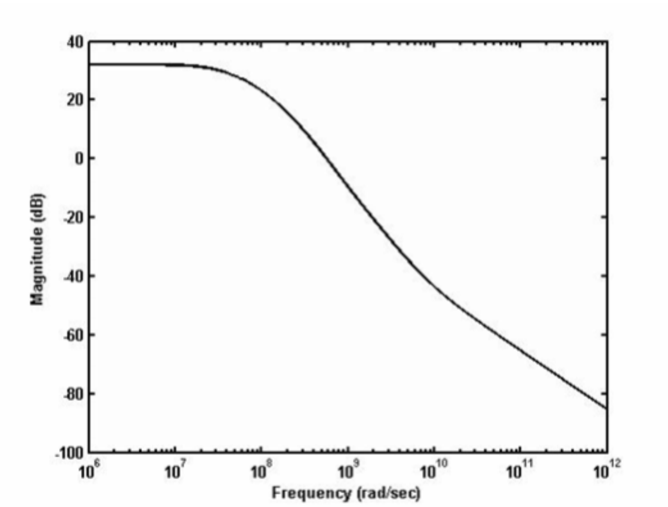

Figure (3). Bode magnitude plot for multistage feedback amplifier.

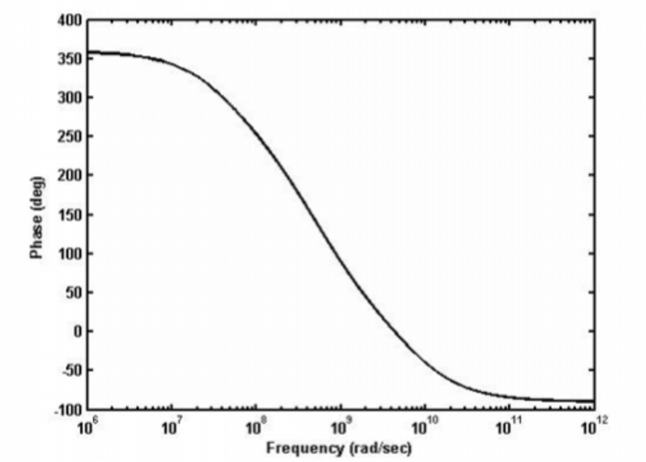

Figure (4). Bode phase plot for multistage feedback amplifier.

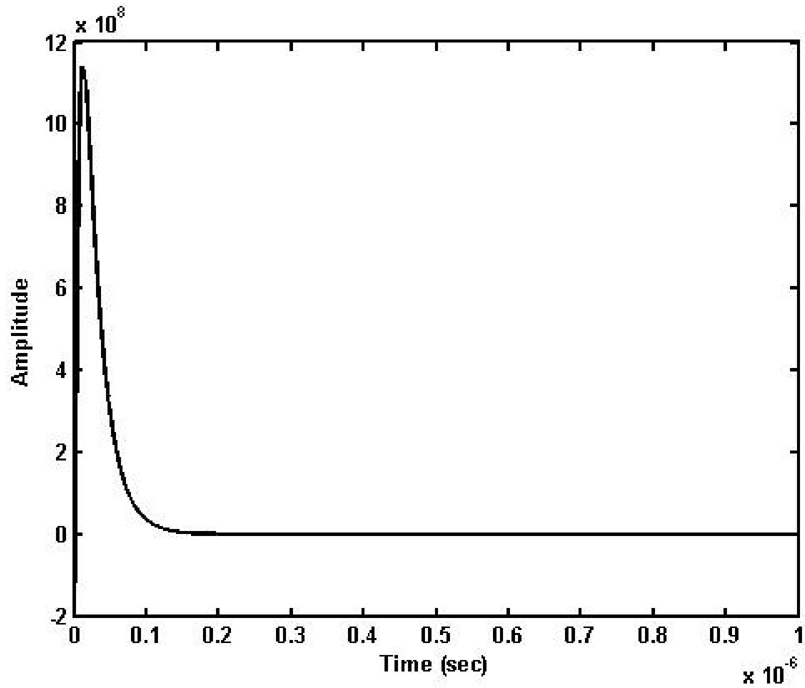

Figure (5). Unit impulse response of the multistage feedback amplifier.

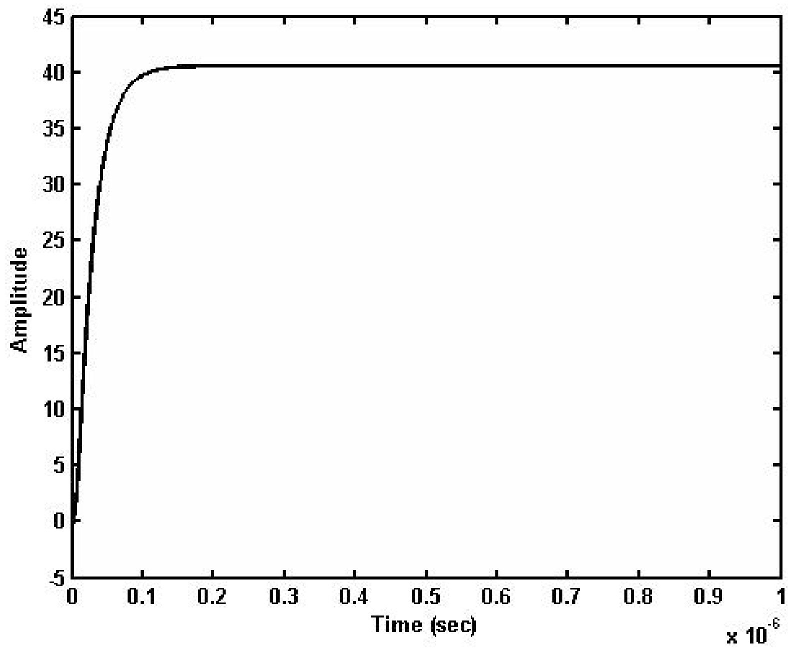

Figure (6). Unit step response of the multistage feedback amplifier.

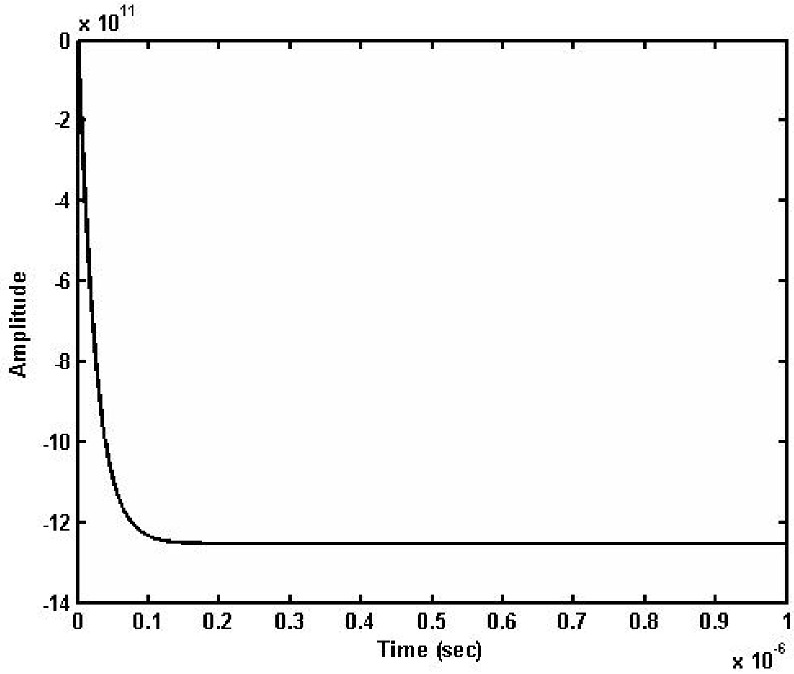

Figure (7). Unit ramp response of the multistage feedback amplifier.

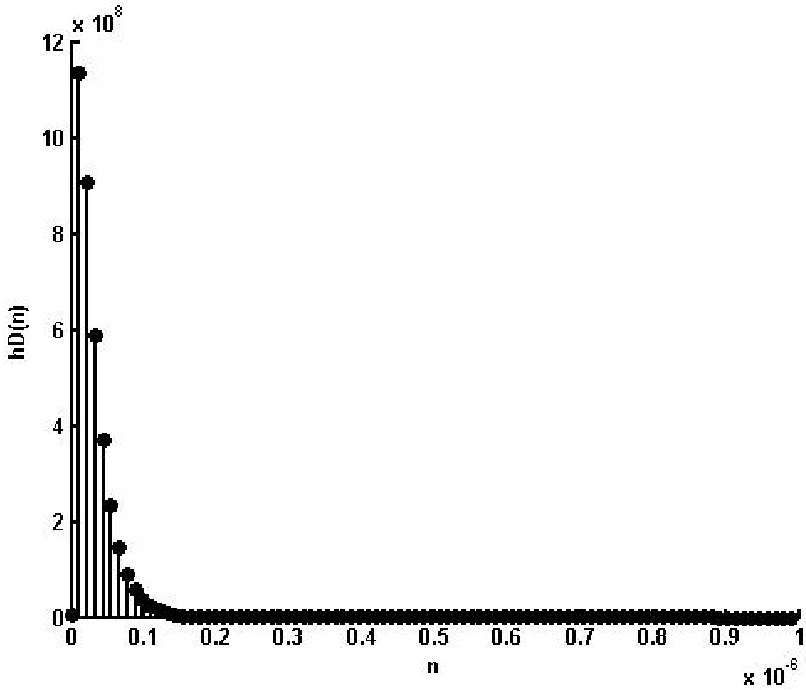

Figure (8). Unit sample response of a discrete-time feedback amplifier.

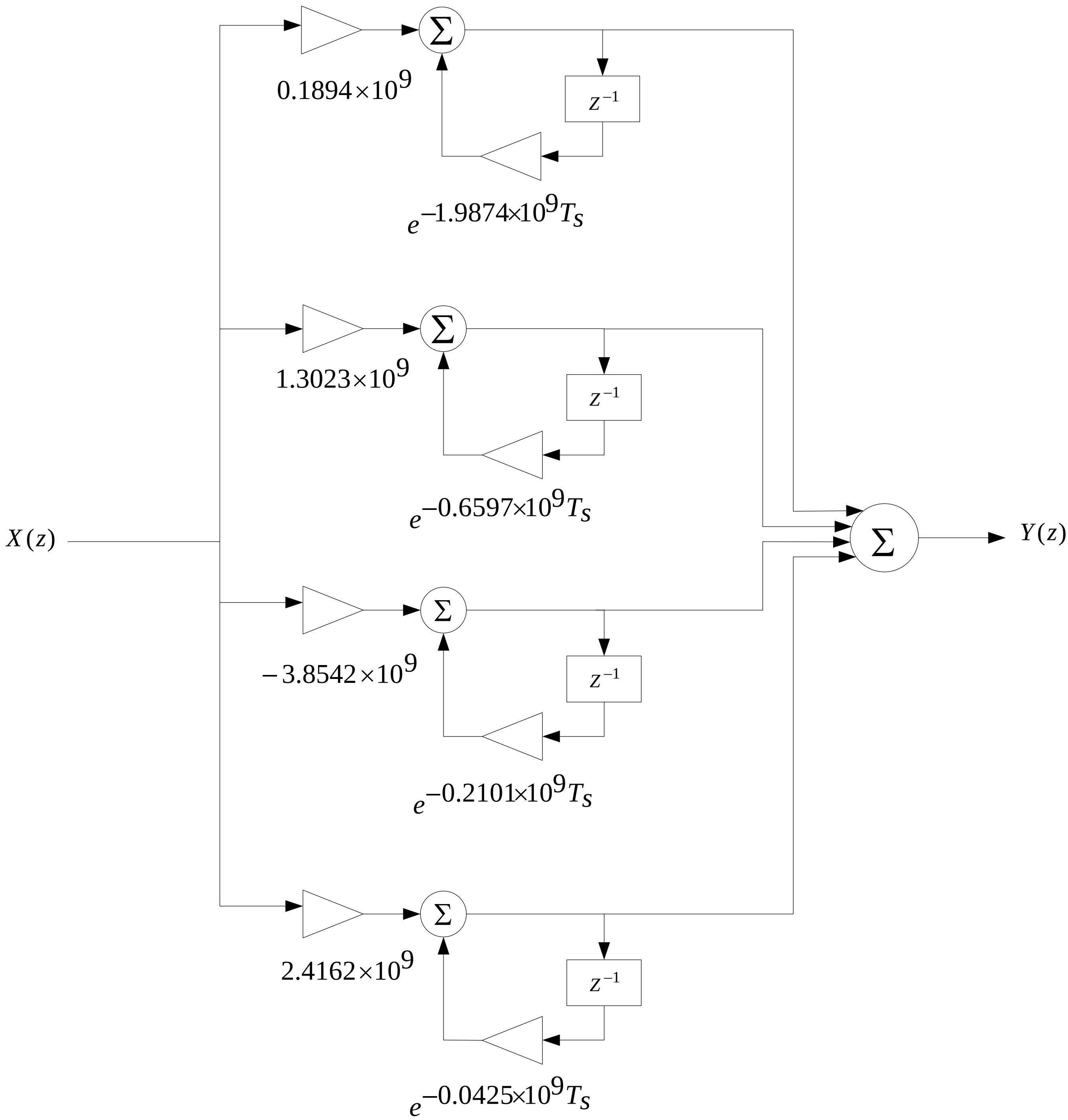

Figure (9). Simulation of an amplifier network by impulse invariant method.

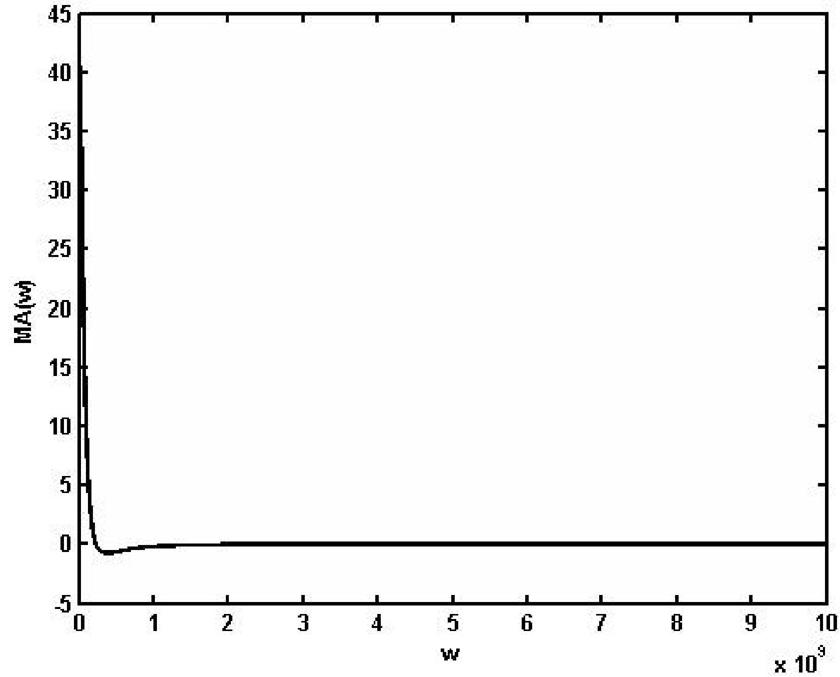

Figure (10). Magnitude response of analogue amplifier.

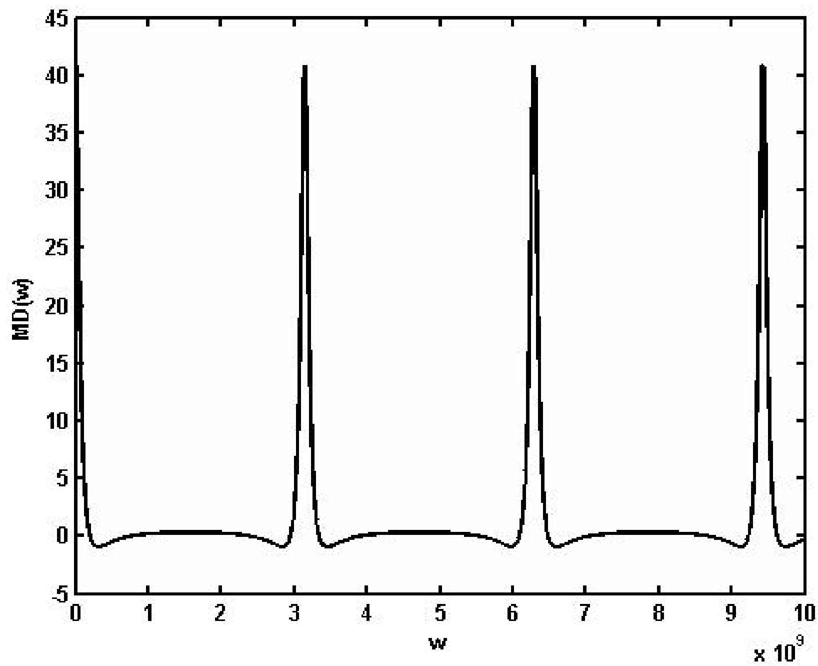

Figure (11). Magnitude response of a discrete-time feedback amplifier.

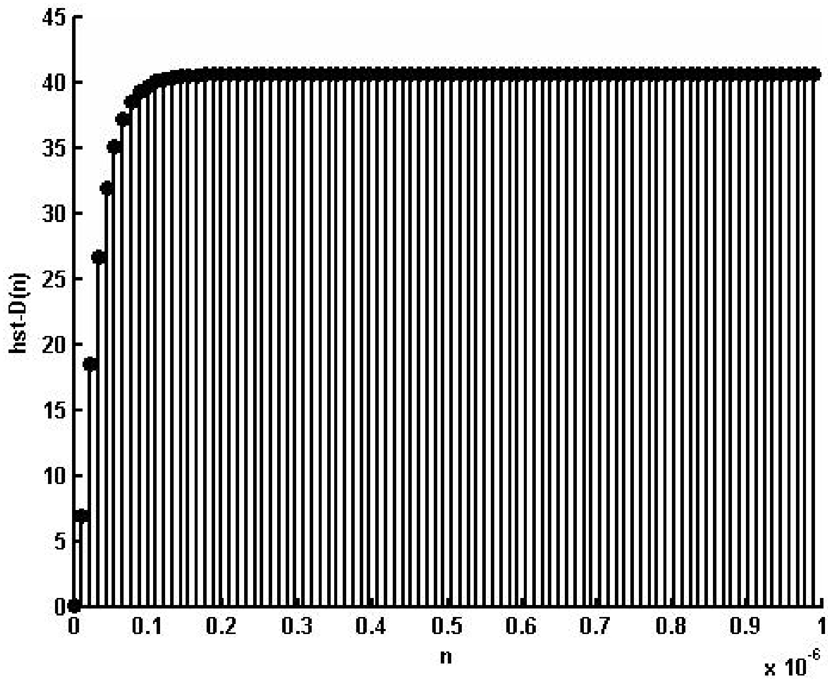

Figure (12). Unit Step sample response of a discrete-time feedback amplifier.

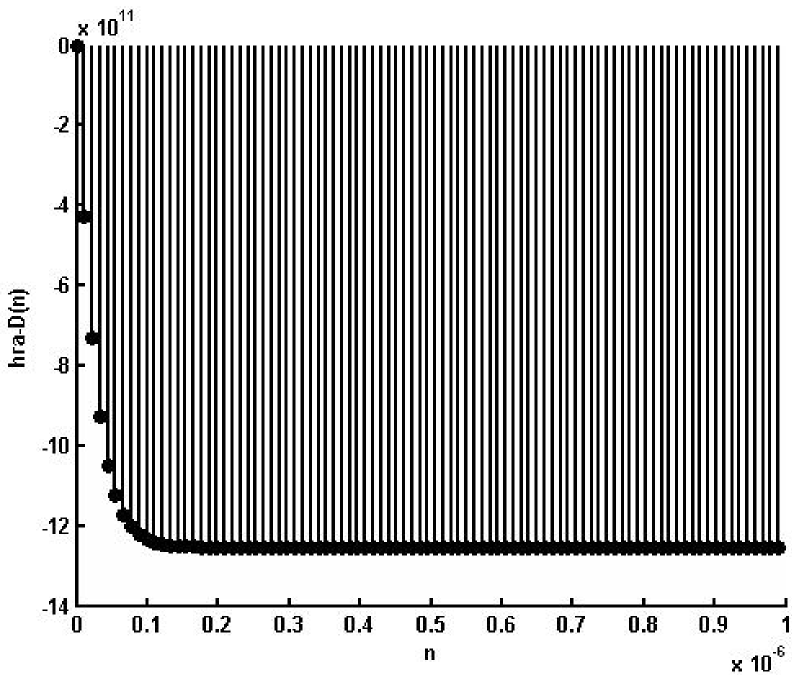

Figure (13). Unit ramp sample response of a discrete-time feedback amplifier.

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